

Effects of non-uniform interstellar magnetic field on synchrotron X-ray and inverse-Compton γ -ray morphology of SNRs

S. Orlando¹, O. Petruk^{2,3}, F. Bocchino¹, and M. Miceli^{4,1}

¹ INAF - Osservatorio Astronomico di Palermo “G.S. Vaiana”, Piazza del Parlamento 1, 90134 Palermo, Italy

² Institute for Applied Problems in Mechanics and Mathematics, Naukova St. 3-b Lviv 79060, Ukraine

³ Astronomical Observatory, National University, Kyryla and Methodia St. 8 Lviv 79008, Ukraine

⁴ Dip. di Scienze Fisiche & Astronomiche, Univ. di Palermo, Piazza del Parlamento 1, 90134 Palermo, Italy

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Abstract

Context. Observations of SNRs in X-ray and γ -ray bands promise to contribute with important information in our understanding of the kinematics of charged particles and magnetic fields in the vicinity of strong non-relativistic shocks and, therefore, on the nature of galactic cosmic rays. The accurate analysis of SNRs images collected in different energy bands requires the support of theoretical modeling of synchrotron and inverse Compton emission from SNRs.

Aims. We develop a numerical code (REMLIGHT) to synthesize, from MHD simulations, the synchrotron radio, X-ray and inverse Compton γ -ray emission originating from SNRs expanding in non-uniform interstellar medium (ISM) and/or non-uniform interstellar magnetic field (ISMF). As a first application, the code is used to investigate the effects of non-uniform ISMF on the SNR morphology in the non-thermal X-ray and γ -ray bands.

Methods. We perform 3D MHD simulations of a spherical SNR shock expanding through a magnetized ISM with a gradient of ambient magnetic field strength. The model includes an approximate treatment of upstream magnetic field amplification and the effect of shock modification due to back reaction of accelerated cosmic rays, assuming both effects to be isotropic. From the simulations, we synthesize the synchrotron radio, X-ray and inverse Compton γ -ray emission with the synthesis code REMLIGHT, making different assumptions about the details of acceleration and injection of relativistic electrons.

Results. A gradient of the ambient magnetic field strength induces asymmetric morphologies in radio, hard X-ray and γ -ray bands independently from the model of electron injection if the gradient has a component perpendicular to the line-of-sight (LoS). The degree of asymmetry of the remnant morphology depends on the details of the electron injection and acceleration and is different in the radio, hard X-ray, and γ -ray bands. In general, the non-thermal X-ray morphology is the most sensitive to the gradient, showing the highest degree of asymmetry. The IC γ -ray emission is weakly sensitive to the non-uniform ISMF, the degree of asymmetry of the remnant morphology being the lowest in this band.

Key words. Magnetohydrodynamics (MHD) – Radiation mechanisms: non-thermal – Shock waves – ISM: supernova remnants – Gamma rays: ISM – X-rays: ISM

1. Introduction

It is largely accepted in the literature that the hard X-ray emission detected in many young shell-type supernova remnants (SNRs) is synchrotron emission from electrons accelerated to energies of tens of TeV (e.g. Koyama et al. 1995) by means of the diffusive shock acceleration process. In addition, inverse Compton (IC) collisions of these high energy electrons with low energy photons from the ambient radiation field (e.g. the cosmic microwave background; hereafter CMB) are expected, thus leading to very-high-energy (VHE; > 100 GeV) γ -ray emission too. In regions of high mass density, ions in the shell are likely to have been accelerated to similar energies, and γ -rays may be due to neutral pion decay from proton-proton interactions. The nature of the TeV emission, therefore, stands on the combination of X-ray synchrotron emitting electrons and very energetic ions; it is not clear, at the present time, which one between the two mainly contributes to the detected VHE γ -rays. The spectral analysis of multi-wavelength data of several shell-type SNRs (Cas A, Aharonian et al.

2001; Albert et al. 2007; RX J1713.7-3946, Muraishi et al. 2000; Enomoto et al. 2002; Aharonian et al. 2007a; Berezhko & Völk 2008; RX J0852.0-4622, Katagiri et al. 2005; Aharonian et al. 2005, 2007b; Enomoto et al. 2006; and RCW 86, Hoppe & Lemoine-Goumard 2008) allows both for leptonic and hadronic origin of VHE γ -rays.

Recently, the study of SNRs as particle accelerators has received a strong impulse thanks to the new γ -ray observations of SNRs with the instruments of the High Energy Spectroscopic System (HESS), the Major Atmospheric Gamma-ray Imaging Cherenkov (MAGIC) experiments and the Fermi Gamma-ray Space Telescope. The analysis of multi-wavelength observations, from radio, to hard X-rays, to γ -rays, promises to increase our understanding of the kinematics of charged particles and magnetic fields in the vicinity of strong non-relativistic shocks and of the possible role of SNRs for the origin of galactic cosmic rays (CRs). In this context, a very important source of information could be the distribution of surface brightness observed in SNRs in several bands. For instance, the properties of the brightness distribution have been crucial in our understanding of the acceleration and injection of relativistic electrons through SNR shocks (e.g. the criterion of Rothenflug et al. 2004 versus the

azimuthal profile comparison in Petruk et al. 2009c). Recently, Petruk et al. (2010a, submitted to MNRAS) have compared synthetic distributions of surface brightness predicted for SN 1006 in different bands with observations, deriving important observational constraints on the modeling of SN 1006. Also, the correlations of brightness in radio, X-rays and γ -rays claimed in RX J1713.7-3946 (Acero et al. 2009) and some others SNRs (e.g. Aharonian et al. 2006) could be considered to favor electrons as being responsible for VHE γ -rays in these SNRs.

The analysis of the brightness distributions observed in different energy bands needs to be supported by an accurate comparison of the observed distributions with those predicted by detailed MHD models. For instance, a number of SNRs present a bilateral structure (BSNRs; Kesteven & Caswell 1987; Fulbright & Reynolds 1990; Gaensler 1998) and it is not clear how to “translate” this 2D information into a 3D morphology of the emission over the SNR shell. The two competing cases traditionally invoked are either equatorial-belt or polar-caps and they are related to the model of injection of relativistic electrons (isotropic or quasi-perpendicular in the former case and quasi-parallel in the latter case). Establishing the 3D morphology of SNRs, therefore, may give some important hints on the acceleration theory.

The distribution of surface brightness of synchrotron emission in SNRs expanding through a uniform interstellar medium (ISM) and uniform interstellar magnetic field (ISMF) has been extensively investigated, through numerical modeling, in both the radio (Fulbright & Reynolds 1990) and X-ray (Reynolds 1998, 2004) bands; in particular, the dependence of the brightness distributions on the efficiency of the acceleration process has been explored, considering different injection models. First IC γ -ray maps of SNRs in a uniform ISM and ISMF have been presented by Petruk et al. (2009a) who investigated the properties of brightness distributions in VHE γ -rays as compared to the distributions in the radio band (see also Petruk et al. 2010b, submitted to MNRAS). Recently, Petruk et al. (2009b) proposed a method to predict IC γ -ray images of SNRs starting from observed synchrotron radio maps and spatially resolved X-ray spectral analysis (e.g. Miceli et al. 2009) of SNRs.

In a previous work (Orlando et al. 2007, hereafter Paper I), we have investigated the origin of asymmetries in the radio morphology of BSNRs through a model of a SNR expanding through either a non-uniform ISM or a non-uniform ISMF. In this paper, we extend our analysis to the non-thermal X-ray and IC γ -ray emission. In particular, we develop a numerical code (REMLIGHT) to synthesize the synchrotron radio, X-ray, and IC γ -ray emission from 3D MHD simulations; then we couple the synthesis code with the MHD model introduced in Paper I (extended to include a simple treatment of upstream magnetic field amplification and the effect of shock modification due to back reaction of accelerated CRs) and investigate the effects of a non-uniform ISMF on the morphology of the remnant in the hard X-ray and γ -ray bands. Though remnants of Type Ia supernovae are expected to expand in an almost uniform ISMF, here we show that even a very small gradient of the ISMF can influence significantly the non-thermal remnant morphology.

In Sect. 2 we describe the MHD model and the numerical setup; in Sect. 3 we describe the computation of synchrotron X-ray and IC γ -ray emission, including model of relativistic electron behavior; in Sect. 4 we discuss the results; and, finally, in Sect. 5 we draw our conclusions.

2. MHD modeling and numerical setup

We adopt the MHD model introduced in Paper I, describing the propagation of a SNR shock through a magnetized ambient medium. The shock propagation is modeled by numerically solving the time-dependent ideal MHD equations of mass, momentum, and energy conservation in a 3D Cartesian coordinate system (x, y, z) (see Paper I for details). The model does not include consistently the effects on shock dynamics due to back-reaction of accelerated CRs. However, we approach the effect of shock modification by considering different values of the adiabatic index γ which is expected to drop from the value of an ideal monoatomic gas; in particular, we consider here the cases of $\gamma = 5/3$ (for an ideal monoatomic gas), $\gamma = 4/3$ (for a gas dominated by relativistic particles), and $\gamma = 1.1$ (for large energy drain from the shock region due to the escape of high energy CRs). In addition, we account for upstream magnetic field amplification due to back reaction of accelerated protons, by amplifying the (pre-shock) ambient magnetic field in the neighborhoods of the remnant. These effects, namely shock modification and magnetic field amplification, might depend on the obliquity angle (i.e. the angle between the external magnetic field and the normal to the shock); for instance, they could follow the same dependence of the injection efficiency (i.e. the fraction of accelerated electrons). However, at the present time, the dependence of these effects on the obliquity angle is not understood and, therefore, we consider here the simplest case of isotropic shock modification and magnetic field amplification (i.e. no additional obliquity-dependent magnetic-field amplification has been assumed). On the other hand, such a choice makes easier our analysis of non-thermal images which are already influenced by the obliquity dependence of other processes (e.g. injection efficiency, magnetic field compression, maximum energy of electrons; see Sect. 3). The simulations are performed using the FLASH code (Fryxell et al. 2000), an adaptive mesh refinement multiphysics code for astrophysical plasmas.

As initial conditions, we adopt parameters appropriate to reproduce the SNR SN 1006 after 1000 yr of evolution: we assume an initial spherical remnant with radius $r_{0\text{SNR}} = 0.5$ pc, originating from a progenitor star with mass of $1.4 M_{\text{sun}}$, and propagating through an unperturbed magneto-static medium. The initial total energy E_0 is set to a value leading to a remnant radius $r_{\text{SNR}} \approx 9$ pc at $t = 1000$ yr (E_0 ranges between ≈ 1.3 and 1.8×10^{51} erg, depending on γ) and is partitioned so that most of the SN energy is kinetic energy. The remnant expands through an homogeneous isothermal medium with particle number density $n = 0.05 \text{ cm}^{-3}$ and temperature $T = 10^4$ K. We consider three different configurations of the unperturbed ambient magnetic field: 1) a uniform ambient magnetic field (runs Unif-g1, Unif-g2, and Unif-g3); 2) a gradient of ambient magnetic field strength perpendicular to the average magnetic field (runs Grad-BZ-g1, Grad-BZ-g2, and Grad-BZ-g3); and 3) a gradient of ambient magnetic field strength aligned with the average magnetic field (runs Grad-BX-g1, Grad-BX-g2, Grad-BX-g3).

In the case of a uniform ISMF, we assume that the field is oriented parallel to the x axis. In the other two cases, the ambient magnetic field is assumed to be dipolar¹. The dipole is oriented parallel to the x axis and located either on the z axis ($x = y = 0$) at $z = -100$ pc (Grad-BZ-g1, Grad-BZ-g2, Grad-BZ-g3) or on the x axis ($y = z = 0$) at $x = -100$ pc (Grad-BX-g1, Grad-BX-g2, Grad-BX-g3). In all the cases, we assume the magnetic field strength to be $B_0 = 30 \mu\text{G}$ at the center of the SN explosion

¹ This idealized situation is adopted here mainly to ensure magneto-staticity of the non-uniform field.

Table 1. Relevant initial parameters of the simulations.

	γ	E_0^a [10^{51} erg]	magnetic field configuration	$(x, y, z)^b$ pc
Unif-g1	5/3	1.30	uniform	-
Unif-g2	4/3	1.54	uniform	-
Unif-g3	1.1	1.81	uniform	-
Grad-BZ-g1	5/3	1.30	z -strat.	(0, 0, -100)
Grad-BZ-g2	4/3	1.54	z -strat.	(0, 0, -100)
Grad-BZ-g3	1.1	1.81	z -strat.	(0, 0, -100)
Grad-BX-g1	5/3	1.30	x -strat.	(-100, 0, 0)
Grad-BX-g2	4/3	1.54	x -strat.	(-100, 0, 0)
Grad-BX-g3	1.1	1.81	x -strat.	(-100, 0, 0)

^a Initial energy of the explosion. ^b Coordinates of the magnetic dipole moment.

($x = y = z = 0$), roughly an order of magnitude higher than the galactic ISMF expected at the location of SN 1006. This high value has been chosen to mimic the effects of upstream magnetic field amplification (see discussion above). In such a way, the post-shock ambient magnetic field is expected to be of the order of a few hundred μ G as deduced from observations. In the configurations with non-uniform ISMF, the field strength varies by a factor ~ 6 over 60 pc: either in the direction perpendicular to the average ambient field $\langle \mathbf{B} \rangle$ (Grad-BZ-g1, Grad-BZ-g2, Grad-BZ-g3), or parallel to $\langle \mathbf{B} \rangle$ (Grad-BX-g1, Grad-BX-g2, Grad-BX-g3). We follow the expansion of the remnant for 1000 yr. Table 1 summarizes the physical parameters characterizing the simulations considered here.

The SN explosion is at the center (x, y, z) = (0, 0, 0) of the computational domain which extends between -10 and 10 pc in all directions. At the coarsest resolution, the adaptive mesh algorithm used in the FLASH code (PARAMESH; MacNeice et al. 2000) uniformly covers the 3D computational domain with a mesh of 8^3 blocks, each with 8^3 cells. We allow for 5 additional nested levels of refinement during the first 100 yr of evolution with resolution increasing twice at each refinement level; then the number of nested levels progressively decreases down to 2 at $t = 1000$ yr as the remnant radius increases following the expansion of the remnant through the magnetized medium. The refinement criterion adopted (Löhner 1987) follows the changes in density and temperature. This grid configuration yields an effective resolution of ≈ 0.0098 pc at the finest level during the first 100 yr of evolution (when the radius of the remnant was < 2 pc) and ≈ 0.078 pc at the end of the simulation, corresponding to an equivalent uniform mesh of 2048^3 and 256^3 grid points, respectively. We assume zero-gradient conditions at all boundaries.

The model does not include the radiative cooling, describing only the free and adiabatic expansion phases of the remnant. The transition time from adiabatic to radiative phase for a SNR is (e.g. Blondin et al. 1998; Petruk 2005)

$$t_{\text{tr}} = 2.84 \times 10^4 E_{51}^{4/17} n_{\text{ism}}^{-9/17} \text{ yr}, \quad (1)$$

where $E_{51} = E_0/(10^{51} \text{ erg})$ and n_{ism} is the particle number density of the ISM. In our set of simulations, $t_{\text{tr}} > 10^5$ yr which is much larger than the time covered by our simulations. Our modeled SNRs therefore never reach the radiative phase. On the other hand, here we aim at describing young SNRs (i.e. those that are still in the adiabatic expansion phase) from which non-thermal X-ray and γ -ray emission is commonly detected.

3. Synchrotron X-ray and inverse-Compton γ -ray emission (REMLIGHT)

From the model results, we synthesize synchrotron radio, X-ray, and IC γ -ray emission, by generalizing the approach of Reynolds (1998) to cases of non-uniform ISM and/or non-uniform ISMF. In Paper I, we have already discussed the synthesis of synchrotron radio emission and we refer the reader to that paper for the details of calculation. Here we discuss the synthesis of X-ray and IC γ -ray emission as it is implemented in the synthesis code REMLIGHT.

One could assume that the synchrotron X-ray or IC γ -ray radiation is due to relativistic electrons distributed with an energy spectrum $N(E) = KE^{-s} \exp(-E/E_{\text{max}})$ electrons $\text{cm}^{-3} \text{ erg s}^{-1}$ (e.g. Gaisser et al. 1998), where E is the electron energy, $N(E)$ is the number of electrons per unit volume with arbitrary directions of motion and with energies in the interval $[E, E + dE]$, K is the normalization of the electron distribution, s the power law index, and E_{max} the maximum energy of electrons accelerated by the shock². Nevertheless, some observations suggest that the cut-off could be broader than pure exponent (e.g. Ellison et al. 2000, 2001; Uchiyama et al. 2003; Lazendic et al. 2004). Therefore we assume that the energy spectrum of electrons is given by

$$N(E) = KE^{-s} \exp \left[- \left(\frac{E}{E_{\text{max}}} \right)^\alpha \right], \quad (2)$$

where $\alpha \leq 1$ is the parameter regulating the broadening of the high-energy end of electron spectrum³.

The volume emissivity due to synchrotron or IC radiation can be expressed as

$$i(\varepsilon) = \int_0^\infty N(E) \Lambda(E, \varepsilon, B) dE, \quad (3)$$

where $\Lambda(E, \varepsilon, B)$ is the radiation power of a single electron with energy E , and ε is the photon energy. The emissivity, $i(\varepsilon)$, depends on the magnetic field strength, B , only in the synchrotron emission process. We compute the surface brightness of the SNR at a given energy ε , by integrating the emissivity $i(\varepsilon)$ at each point along each LoS in a raster scan (assuming that the source is optically thin).

In the case of synchrotron emissivity in the X-ray band, $i_X(\varepsilon)$, the spectral distribution of radiation power of a single electron with energy E in the magnetic field \mathbf{B} is

$$\Lambda_X(E, \varepsilon) = \frac{\sqrt{3}e^3\mu_\phi B}{m_e c^2} F\left(\frac{\varepsilon}{\varepsilon_c}\right), \quad (4)$$

where $\varepsilon_c = h\nu_c = hc_1\mu_\phi BE^2$, h is the Planck constant, ν_c is the critical frequency, ϕ the angle between the magnetic field and the LoS, μ_ϕ is either $\mu_\phi = \sin \phi$ for the case of ordered magnetic field or $\mu_\phi = \langle \sin \phi \rangle = \pi/4$ for disordered magnetic field, $c_1 = 3e/(4\pi m_e^3 c^5)$, e and m_e are the charge and mass of electron, respectively, c is the speed of light. The special function $F(w)$

² Note that the distribution of electrons $N(E)$ in the case of synchrotron radio emission is expressed as $N(E) = KE^{-s}$ electrons $\text{cm}^{-3} \text{ erg s}^{-1}$ (see, for instance, Paper I).

³ Note however that recent work by Kang & Ryu (2010) suggests that $\alpha > 1$.

can be approximated as (e.g. Rybicki & Lightman 1985; Wallis 1959):

$$F(w) = \begin{cases} 2.15 w^{1/3} & w < 0.01, \\ \sqrt{\pi} w^{0.29} \exp(-w) & 0.01 \leq w \leq 5, \\ \sqrt{\pi/2} w^{1/2} \exp(-w) & w > 5. \end{cases} \quad (5)$$

We found the above approximation quite accurate with discrepancies $\lesssim 4\%$ from the exact value. In addition, $\int_0^\infty F d\varepsilon = 1.59$ while the exact value is $8\pi/9 \sqrt{3} = 1.61$.

In the case of γ -ray emissivity due to IC process, i_{IC} , the spectral distribution $\Lambda_{IC}(E, \varepsilon)$ of radiation power of a single electron in a black-body photon field in Eq. 3 is (see also Petruk et al. 2009a)

$$\Lambda_{IC}(E, \varepsilon) = \frac{2e^4 \epsilon_c}{\pi \hbar^3 c^2} \Gamma^{-2} \mathcal{I}_{ic}(\eta_c, \eta_0), \quad (6)$$

where Γ is the Lorentz factor of electron, $\epsilon_c = kT_{CMBR}$, T_{CMBR} is the temperature of the cosmic microwave background radiation (CMBR) assumed to be $T_{CMBR} = 2.75$ K,

$$\eta_c = \frac{\epsilon_c \varepsilon}{(m_e c^2)^2}, \quad \eta_0 = \frac{\varepsilon^2}{4\Gamma m_e c^2 (\Gamma m_e c^2 - \varepsilon)}, \quad (7)$$

and the special function $\mathcal{I}_{ic}(\eta_c, \eta_0)$ can be accurately approximated as (Petruk 2009)

$$\mathcal{I}_{ic}(\eta_c, \eta_0) \approx \frac{\pi^2}{6} \eta_c \left\{ \exp \left[-\frac{5}{4} \left(\frac{\eta_0}{\eta_c} \right)^{1/2} \right] + 2\eta_0 \exp \left[-\frac{5}{7} \left(\frac{\eta_0}{\eta_c} \right)^{0.7} \right] \right\} \exp \left[-\frac{2}{3} \left(\frac{\eta_0}{\eta_c} \right) \right]. \quad (8)$$

This approximation represents $\mathcal{I}_{ic}(\eta_c, \eta_0)$ in any regime, from Thomson to extreme Klein-Nishina. The approximation is exact in the Thomson limit. It restores detailed calculations with maximum error of 30% in the range of parameters which gives non-negligible contribution to emission.

3.1. Maximum energy of electrons

We follow the approach of Reynolds (1998) for the description of time evolution and surface variation of E_{\max} , generalizing his approach to cases of non-uniform ISM and/or non-uniform ISMF. Reynolds (1998) considered three alternatives for time and spatial dependence of E_{\max} . Namely, the maximum accelerated energy maybe determined: 1) by the electron radiative losses (due to synchrotron and IC processes), 2) by the limited time of acceleration (due to the finite age of the remnant) and 3) by properties of micro-physics when the scattering of electrons with $E > E_{\max}$ becomes less efficient and the electrons freely escape from the region of acceleration⁴. The maximum energy is given by

$$E_{\max, \xi} \propto f_{E, \xi}(\Theta_0) V_{sh}^{q_\xi} B_o^{\lambda_\xi}, \quad (9)$$

⁴ For the escape case, it is commonly assumed that MHD waves responsible for the scattering are much weaker above some wavelength, λ_{\max} , and E_{\max} is approximately the energy of particles with that gyro-radius (e.g. Reynolds 1998).

where Θ_0 is the obliquity angle, $f_{E, \xi}(\Theta_0)$ is a function describing smooth variations of E_{\max} versus obliquity, V_{sh} is the shock velocity, B_o is the pre-shock ISMF strength, and $\xi = 1, 2, 3$ corresponds respectively to loss-limited, time-limited and escape-limited models of E_{\max} . The values of q and λ are: $q_1 = 1$, $q_2 = q_3 = 0$, and $\lambda_1 = -1/2$, $\lambda_2 = \lambda_3 = 1$ (Reynolds 1998). Note that we assume $q_2 = 0$ because 1) E_{\max} rises quite slowly with time when it is determined by the finite time of acceleration (Reynolds 1998), even in the nonuniform ISM, and 2) most of emission rises from downstream regions close to the shock; therefore, the variation of E_{\max} due to velocity variation is negligible in the (thin) emitting region. From Eq. 9, we express the surface variation of E_{\max} as

$$E_{\max, \xi} = E_{\max, \xi, ||} f_{E, \xi}(\Theta_0) \mathcal{V}_{sh}^{q_\xi} \mathcal{B}_o^{\lambda_\xi}, \quad (10)$$

where $E_{\max, \xi, ||}$ is a free parameter, representing the maximum energy in a point p on the SNR surface where the ISMF is parallel to the shock normal, $\mathcal{V}_{sh} = V_{sh}/V_{sh, ||}$, $\mathcal{B}_o = B_o/B_{o, ||}$, $V_{sh, ||}$ and $B_{o, ||}$ are the shock velocity and pre-shock ISMF strength in the point p , respectively.

A detailed theoretical framework providing the obliquity dependence of E_{\max} was presented by Reynolds (1998) and is based on the prescription for diffusion by Jokipii (1987). However, this theory is limited to the test-particle regime, assuming no magnetic field amplification. On the other hand, at the present time, a more general theory describing the obliquity dependence of E_{\max} is still lacking. For the sake of generality, we adopt here some arbitrary smooth variations of E_{\max} versus obliquity with the goal to see how different trends in the obliquity dependence of E_{\max} influence the visible morphology of SNRs. In fact, the remnant morphology – once we are not interested in the exact comparison with observations – is mainly affected by the contrast $C_{\max} = E_{\max, ||}/E_{\max, \perp}$ and not by the exact shape of the dependence on obliquity, once the latter is assumed to be smooth.

Our strategy is to consider smooth variations of E_{\max} versus obliquity that correspond to the loss-limited, time-limited and escape-limited models of E_{\max} in the theoretical framework of Reynolds (1998). Since, in general, the mechanism limiting electron acceleration (i.e. loss-limited, time-limited and escape-limited) may be different at different shock obliquity angles, we calculate the value of E_{\max} appropriate for each limitation mechanism at each point, by considering $E_{\max} = \min[E_{\max, 1}, E_{\max, 2}, E_{\max, 3}]$ (where indexes 1, 2, 3 correspond respectively to loss-limited, time-limited and escape-limited models). This way to compute E_{\max} is adopted in Sect. 4.2, 4.3, and 4.6 where we assume also to be in the Bohm limit (i.e. gyrofactor⁵ $\eta = 1$) in the test-particle regime. In particular, in Sect. 4.2, we introduce a reference case for which the adopted set of parameters (see Sect. 4.1) leads to $C_{\max} > 1$. Note that the adopted parameters make this case suitable for comparison with young non-thermal SNRs as, for instance, SN 1006. In addition, for the sake of generality, in Sect. 4.4 and 4.5, we explore the effects on the remnant morphology of various obliquity dependencies of E_{\max} , by considering also cases for which the contrast C_{\max} is < 1 .

⁵ The “gyrofactor” is defined as the ratio between the mean free path, $\lambda_{||}$, along the magnetic field and the gyroradius, r_g (see Reynolds 1998). In general it is expected that the mean free path can be no less than r_g , so that $\eta \geq 1$; the equality corresponds to the Bohm limit, i.e. a level of turbulence leading to wave amplitudes comparable to the stationary magnetic field strength.

3.2. Post-shock evolution of the electron distribution

As in Reynolds (1998), we assume that relativistic electrons are confined in the fluid elements which advect them from the region of acceleration. Fluid element with Lagrangian coordinate $\mathbf{a} \equiv \mathbf{R}(t_i)$ was shocked at time t_i , where R is the radius of the shock. At that time, the electron distribution on the shock was

$$N(E_i, t_i) = K_s(a, t_i) E_i^{-s} \exp \left[- \left(\frac{E_i}{E_{\max}(t_i)} \right)^\alpha \right], \quad (11)$$

where E_i is the electron energy at time t_i , K_s is the normalization of the electron distribution immediately after the shock (in the following, index “s” refers to the immediately post-shock values), and s is the power law index. At variance with Paper I, we are interested here in synchrotron X-ray and IC γ -ray emission. In this case, the evolution of the electron distribution has to account for energy losses of electrons due to both adiabatic expansion and radiative losses caused by synchrotron and IC processes. At time t_i , the energy of the electron confined in the fluid element with Lagrangian coordinates $\mathbf{a} \equiv \mathbf{R}(t_i)$ was (cf. Eq. 26 in Reynolds 1998)

$$E_i = \frac{E}{\mathcal{E}_{\text{ad}} \mathcal{E}_{\text{rad}}} \quad (12)$$

where E is the electron energy at the present time t , \mathcal{E}_{ad} is a term accounting for the energy losses of electrons due to adiabatic expansion

$$\mathcal{E}_{\text{ad}}(a, t) = \left(\frac{\rho(a, t)}{\rho_s(t_i)} \right)^{1/3} = \left(\frac{\rho(a, t)}{\rho_s(t)} \right)^{1/3} \left(\frac{\rho_o(R)}{\rho_o(a)} \right)^{1/3} \quad (13)$$

ρ is the mass density (in the following, index “o” refers to the pre-shock values), \mathcal{E}_{rad} is a term accounting for the radiative losses of electrons

$$\mathcal{E}_{\text{rad}}(E, a, t) = 1 - I(a, t) \frac{E}{E_{f, \parallel}} \quad (14)$$

$E_{f, \parallel} = 637 / (B_{\text{eff}, s, \parallel}^2 t)$ erg is the fiducial energy at parallel shock (Reynolds 1998), $B_{\text{eff}, s, \parallel}^2 = B_{\parallel}^2 + B_{\text{CMB}}^2$ is an “effective” magnetic field at parallel shock accounting energy losses due to IC scatterings on the photons of CMB, $B_{\text{CMB}} = 3.27 \mu\text{G}$ is the magnetic field strength with energy density equal to that in the CMB, t is the time, and $I(a, t)$ is an integral independent of E which is calculated with the approach described in Appendix A. The electron energy losses in a given fluid element are mainly due to radiative losses if $E_f < E_{\max}$ and to adiabatic expansion if $E_f \gtrsim E_{\max}$.

At time t_i , the shock was able to accelerate an electron confined in the fluid element with Lagrangian coordinate a to $E_{\max}(t_i)$. From Eq. 9, we derive that

$$\frac{E_{\max}(t_i)}{E_{\max}(t)} = \frac{f_E(\Theta_o(t_i))}{f_E(\Theta_o(t))} \left(\frac{V_{\text{sh}}(t_i)}{V_{\text{sh}}(t)} \right)^q \left(\frac{B_o(a)}{B_o(R)} \right)^\lambda \equiv \mathcal{F}(a, R). \quad (15)$$

The ratio, $V_{\text{sh}}(t_i)/V_{\text{sh}}(t)$, may be expressed through pressure, P , and density, ρ (Hnatyk & Petruk 1999)

$$\frac{V_{\text{sh}}(t_i)}{V_{\text{sh}}(t)} = \left(\frac{P(a, t)}{P_s(t)} \right)^{1/2} \left(\frac{\rho_o(a)}{\rho_o(R)} \right)^{(\gamma-1)/2} \left(\frac{\rho(a, t)}{\rho_s(t)} \right)^{-\gamma/2}. \quad (16)$$

The conservation law for the number of particles per unit volume per unit energy interval

$$N(E, a, t) = N(E_i, a, t_i) \frac{a^2 da dE_i}{\sigma r^2 dr dE}, \quad (17)$$

where $\sigma = (\gamma + 1)/(\gamma - 1)$ is the shock compression ratio and r is the Eulerian coordinate, together with the continuity equation $\rho_o(a) a^2 da = \rho(a, t) r^2 dr$ and the derivative

$$\frac{dE_i}{dE} = \frac{1}{\mathcal{E}_{\text{ad}} \mathcal{E}_{\text{rad}}^2}, \quad (18)$$

implies that downstream

$$N(E, a, t) = K(a, t) E^{-s} \mathcal{E}_{\text{rad}}^{s-2} \times \exp \left[- \left(\frac{E}{E_{\max}(t, \Theta_o) \mathcal{F}(a, R) \mathcal{E}_{\text{ad}} \mathcal{E}_{\text{rad}}} \right)^\alpha \right], \quad (19)$$

with $K(a, t) = K_s(t_i) \mathcal{E}_{\text{ad}}^{s+2}$, $E_{\max}(t, \Theta_o)$ is given by Eq. 10, and taking into account Eq. 15. Assuming that $K_s \propto \rho_s V_{\text{sh}}(t)^{-b}$, i.e. it varies with the shock velocity $V_{\text{sh}}(t)$ and, in case of non-uniform ISM, with the immediately post-shock value of mass density, ρ_s , the downstream variation of $K(a, t)$ is described by the relation (see Paper I)

$$\frac{K(a, t)}{K_s(R, t)} = \frac{f_\zeta(\Theta_o(t_i))}{f_\zeta(\Theta_o(t))} \left(\frac{P(a, t)}{P_s(R, t)} \right)^{-b/2} \times \left(\frac{\rho_o(a)}{\rho_o(R)} \right)^{-b(\gamma-1)/2 - (s-1)/3} \left(\frac{\rho(a, t)}{\rho_s(R, t)} \right)^{b\gamma/2 + (s+2)/3}, \quad (20)$$

where $f_\zeta(\Theta_o)$ is the obliquity dependence of the injection efficiency ζ (the fraction of accelerated electrons). Again, in the lack of theoretical dependence of the injection efficiency on obliquity in case of efficient acceleration, we consider a number of smooth functions for $f_\zeta(\Theta_o)$, exploring either increasing or decreasing injection. In fact, such an approach, which considers different contrasts $f_{\zeta \parallel} / f_{\zeta \perp}$ either > 1 or < 1 is realized in our previous paper Petruk et al. (2009a); there, it was shown how the change in the injection contrast influences non-thermal images of SNRs in uniform ISM and uniform ISMF. In the present paper, for simplicity and in order to see the effects from nonuniformity of ISMF, we limit our considerations to three contrasts of injection. Namely, following Reynolds (1998), we consider the following models: quasi-parallel ($f_\zeta(\Theta_o) = \cos^2 \Theta_s$), isotropic ($f_\zeta(\Theta_o) = 1$), and quasi-perpendicular ($f_\zeta(\Theta_o) = \sin^2 \Theta_s$), where Θ_s is related to Θ_o through the expression $\cos \Theta_s = \sigma_B^{-1} \cos \Theta_o$. The first injection model leads to a three-dimensional polar-caps structure of the remnant, whereas the latter two produce a three-dimensional equatorial-belt structure of the remnant.

Radiative losses of electrons $\dot{E} \propto E^2$ are mostly effective in modification of the distribution $N(E, a, t)$ around $E \sim E_{\max}$ (Reynolds 1998). This may be noted in Eq. 19. The variation of the energy distribution $N(E, a, t)$ of electrons with energy $E \ll E_{\max}$ (in this case also $E \ll E_f$, leading to $\mathcal{E}_{\text{rad}} \rightarrow 1$), i.e. electrons with negligible radiative losses, is given by $N(E, a, t)/N_s(E, R, t) = K(a, t)/K_s(R, t)$, where $N_s(E, R, t)$ is the energy distribution of electrons immediately after the shock. This expression does not depend on energy E and, in fact, we have used this expression in Paper I for investigation of properties of surface brightness distribution of SNR emitting radio frequencies. In contrast, the modification of the distribution $N(E, a, t)$ due to effective electron radiation is given by the two last multipliers in Eq. 19. The radiative losses of electrons therefore are important for the surface brightness distribution of SNR in X-ray and γ -rays.

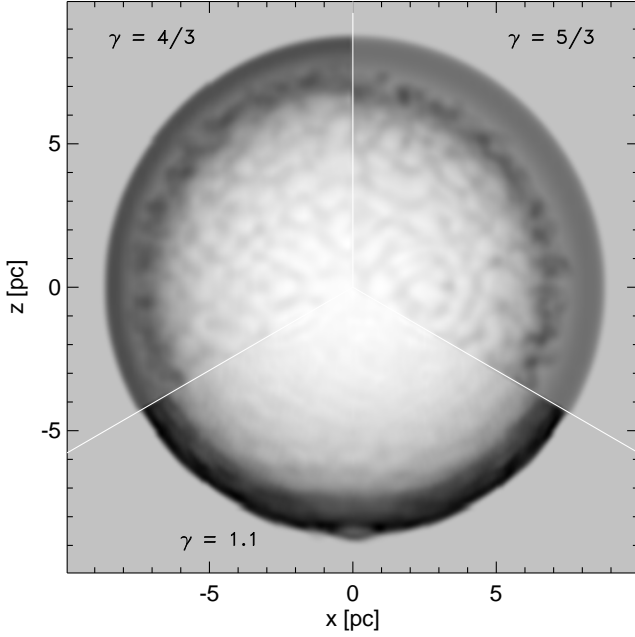


Figure 1. 3D rendering of the mass density at $t = 1000$ yr for a remnant expanding through a uniform ISMF and for three cases of $\gamma = 5/3, 4/3, 1.1$ (runs Unif-g1, Unif-g2, and Unif-g3, see Table 1).

4. Synchrotron and IC images of SNRs expanding through non-uniform ISMF

The evolution of the remnant expanding through the non-uniform ISMF has been described in Paper I where the reader is referred to for more details. Figure 1 shows the 3D rendering of the mass density at $t = 1000$ yr in the three cases of γ considered for uniform ISMF. The main effect of γ on the shock dynamics is to change its compression ratio and the distance of the contact discontinuity from the blast wave position; no dependence on the obliquity angle is present, γ being uniform in each simulation. The value of γ is expected therefore to influence the absolute values of emission in the radio, X-ray and γ -ray bands but not the large scale morphology of the remnant to which this paper is focused on. In the following, we first discuss the effects of non-uniform ISMF on the synchrotron and IC emission adopting, as reference, the case with $\gamma = 5/3$, allowing the direct comparison of our results with those available in the literature; then in Sect. 4.3, we discuss the effect of γ on the morphology of the non-thermal emission.

In all the synthetic images presented below, we introduce the procedure of magnetic field disordering (with randomly oriented magnetic field vector in each point) downstream of the shock (see Paper I), according to observations showing a low degree of polarization (10-15%; e.g. Tycho, Dickel et al. 1991, SN 1006, Reynolds & Gilmore 1993). Note that the obliquity angle Θ_s is derived from Θ_0 through the conversion formula $\cos \Theta_s = \sigma_B^{-1} \cos \Theta_0$ given in Sect. 3.2 and, therefore, does not take into account the magnetic field disordering; as discussed by Fulbright & Reynolds (1990), this corresponds to the assumption that the disordering process takes place over a longer time-scale than the electron injection which occurs in the close proximity of the shock.

In all the simulations, we assume the (average) unperturbed ISMF $\langle \mathbf{B} \rangle$ oriented along the x axis. In the two magnetic field

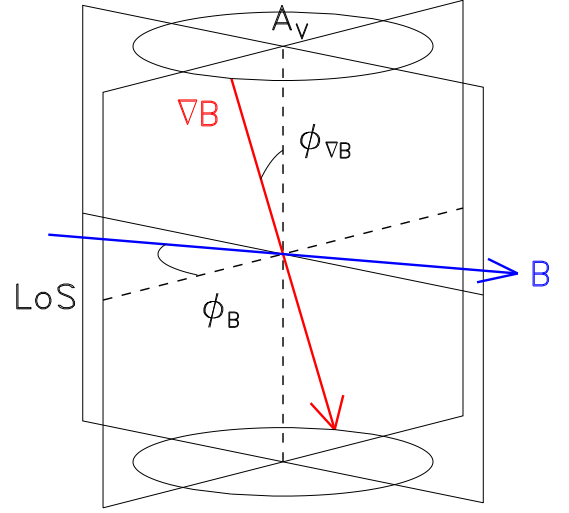


Figure 2. Relevant angles describing the orientation of the ISMF and of the gradient of ISMF strength with respect to the observer: ϕ_B is the angle between the (average) unperturbed ISMF and the LoS, and $\phi_{\nabla B}$ is the angle between the gradient of the ISMF strength and the vertical line passing through the center of the remnant A_v .

configurations explored in this paper, the gradient of ISMF strength is either normal (runs Grad-BZ-g1, Grad-BZ-g2, Grad-BZ-g3; $\nabla|\langle \mathbf{B} \rangle|$ along z) or aligned (runs Grad-BX-g1, Grad-BX-g2, Grad-BX-g3; $\nabla|\langle \mathbf{B} \rangle|$ along x) to $\langle \mathbf{B} \rangle$. Since we analyze the remnant morphology as it would be observed from different points of view, we define two angles to describe the orientation of $\langle \mathbf{B} \rangle$ and $\nabla|\langle \mathbf{B} \rangle|$ in the space (see Fig. 2): ϕ_B is the angle between $\langle \mathbf{B} \rangle$ and the LoS, and $\phi_{\nabla B}$ is the angle between $\nabla|\langle \mathbf{B} \rangle|$ and the normal to the ISMF in the plane of the sky (axis A_v in Fig. 2). The first angle is the aspect angle commonly used in the literature. The definition of the second angle allows us to explore the remnant morphology for various aspect angles and for fixed $\phi_{\nabla B}$, $\nabla|\langle \mathbf{B} \rangle|$ lying on a cone with angle $\phi_{\nabla B}$ (see Fig. 2). In cases in which the gradient ∇B is aligned with the average ISMF (runs Grad-BX-g1, Grad-BX-g2, Grad-BX-g3) $\phi_{\nabla B} = 90^\circ$ by definition. In Grad-BZ models, the angle between $\langle \mathbf{B} \rangle$ and $\nabla|\langle \mathbf{B} \rangle|$ is always 90° . In the following, the images are calculated for various values of the angles defined above and with a resolution of 256×256 pixels.

4.1. Parameter space

The prescriptions for the electron energy distribution at any point inside the remnant and for the synthesis of synchrotron and IC emission discussed in Sect. 3 are characterized by several parameters regulating the energy spectrum of relativistic electrons, the injection efficiency, the time and spatial dependence of E_{\max} , etc.. In the following, we limit the model parameter space through some assumptions that allow us to fix some of the parameters.

In particular, we assume that the power law index in Eq. 2 is $s = 2$, as suggested by many observations of BSNRs (e.g. for SN 1006; Miceli et al. 2009). In the test-particle regime, the index should be related to the shock compression ratio through $s = (2 + \sigma)/(\sigma - 1)$ with $\sigma = (\gamma + 1)/(\gamma - 1)$. In case of efficient shock acceleration, the electron energy distribution is curved. The value of s that should be used in an approximation like Eq. 2

is rather a mean radio-to-X-ray spectral index whose value is around 2 (e.g. Allen et al. 2008), independent of the local slopes of the electron spectrum. As for the curvature of the spectrum around E_{\max} , we assume that $\alpha = 0.5$ in Eq. 2. In fact, this could be the case in SN 1006 and G347.3-0.5 where a number of models suggest $\alpha \approx 0.5$ (Ellison et al. 2000, 2001; Uchiyama et al. 2003; Lazendic et al. 2004).

The maximum energy at parallel shock $E_{\max,\parallel}$ is a free parameter in Eq. 10 that we assume to be $E_{\max,\parallel} = 26$ TeV in most of our calculations. This parameter has to be compared with the fiducial energy at parallel shock defined as $E_{f,\parallel} = 637/(B_{\text{eff},s,\parallel}^2 t)$ erg (Reynolds 1998); for the cases considered here, we have $E_{f,\parallel} = 14$ TeV in models with uniform ISMF, $E_{f,\parallel} = 12$ TeV in Grad-BZ models and $E_{f,\parallel} = 2$ TeV in Grad-BX models⁶. In all these cases, therefore, $E_f < E_{\max}$ for a significant portion of the remnant and the electron energy losses are mainly due to radiative losses (see discussion in Sect. 3.2). In Sect. 4.6, we investigate the dependence of the non-thermal emission on $E_{\max,\parallel}$, by exploring cases for which $E_{\max} < E_f$ and the electron energy losses are mainly due to adiabatic expansion.

The parameter b in Eq. 20 is a constant and determines how the injection efficiency depends on the shock properties; we assumed that $K_s \propto \rho_s V_{\text{sh}}(t)^{-b}$ (see Sect. 3.2 and Paper I). On theoretical grounds b might be expected to be negative, reflecting an expectation that injection efficiency may behave in a way similar to acceleration efficiency: stronger shocks might inject particles more effectively. Reynolds (1998) considered three empirical alternatives for b as a free parameter, namely, $b = 0, -1, -2$. In particular, $b = -2$ is commonly assumed in many areas of astrophysics such as gamma-ray bursts and prompt radio and X-ray emission from SNe. However Bandiera & Petruk (2010) have shown that models preferring a constant fraction of the shock energy to be transferred into CRs (i.e. $b = -2$) are rejected by statistical analysis of two SNR samples. In addition, Petruk et al. (2010a, submitted to MNRAS) compared their model results with experimental data of the remnant SN 1006 and found that b has a value between 0 and -1 . Petruk et al. (2010b, submitted to MNRAS) showed that the smaller b , the thicker the radial profiles of the surface brightness in all bands; an effect mostly prominent in radio band. Since no effect on the pattern of asymmetries induced by a non-uniform ISMF is expected (see Paper I), we assume $b = 0$ in all our calculations, this being the most neutral case.

The synthetic images are expected to depend on the remnant age (Reynolds 1998). To reduce further the number of model parameters, we focus here on remnant 1 kyr old, as in the case of SN 1006. Finally, radio, X-ray and γ -ray images are synthesized at 1 GHz, 3 keV, and 1 TeV, respectively. It is worth to emphasize that all the above parameters are not expected to influence the pattern of the asymmetries induced by a non-uniform ISMF on which the present paper is focused (see, also, Sect. 4.6 for a discussion on the influence of $E_{\max,\parallel}$ on the remnant morphology).

4.2. Asymmetries in the remnant morphology: the reference case

In Paper I, we analyzed the asymmetries induced by a non-uniform ISMF in the radio morphology of the remnant. In par-

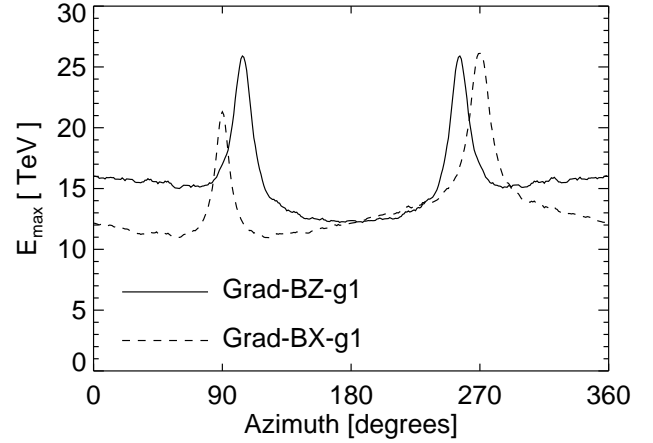


Figure 3. Azimuthal profiles of the maximum energy E_{\max} computed in run Grad-BZ-g1 (solid line) and Grad-BX-g1 (dashed line) when the aspect angle is $\phi_B = 90^\circ$ and the gradient of magnetic field strength lies in the plane of the sky ($\phi_{\nabla B} = 0^\circ$ in run Grad-BZ-g1 and $\phi_{\nabla B} = 90^\circ$ in run Grad-BX-g1). The shock is parallel around 90° and 270° and perpendicular around 180° and 360° . In both models, the contrast $C_{\max} > 1$.

ticular, we found there that asymmetric BSNRs are produced if a gradient of the ambient magnetic field strength ∇B is not aligned with the LoS. In this section we extend our analysis to non-thermal X-rays and IC γ -rays. To this end, we synthesize the synchrotron and IC emission, considering each of the three cases of variation of electron injection efficiency with shock obliquity (quasi-perpendicular, isotropic, and quasi-parallel particle injection). Also we assume the adiabatic index to be $\gamma = 5/3$; the effects of lower γ values on the remnant morphology are explored in Sect. 4.3. Here the maximum energy of electrons is calculated at each point as $E_{\max} = \min[E_{\max,1}, E_{\max,2}, E_{\max,3}]$ (where indexes 1, 2, 3 correspond respectively to loss-limited, time-limited and escape-limited models; see discussion in Sect. 3.1). For the set of parameters chosen for our simulations, it turns out that the loss-limited model is dominant at all obliquity angles, thus simplifying the analysis of non-thermal images in this section. Figure 3 shows E_{\max} versus the azimuthal angle (the azimuth is measured counterclockwise from the "north" of the remnant) for runs Grad-BZ-g1 and Grad-BX-g1. In both cases, E_{\max} is characterized by two maxima where the ISMF is parallel to the shock normal (around 90° and 270°); thus the contrast of E_{\max} is $C_{\max} > 1$. The strength of the unperturbed ISMF is the largest at 180° (90°) and the lowest at 360° (270°) in run Grad-BZ-g1 (Grad-BX-g1) due to the magnetic field gradient. The latter determines the asymmetries in the azimuthal profile of E_{\max} : in run Grad-BZ-g1, the two maxima are converging on the side where the field is the most intense, the gradient being perpendicular to the average magnetic field; in run Grad-BX-g1, the two maxima have different intensities (with the largest where the magnetic field is the lowest⁷), the gradient being parallel to the average magnetic field.

As an example, Figs. 4 and 5 show the maps of synchrotron radio, X-ray, and IC γ -ray surface brightness at $t = 1$ kyr, in each of the three injection models (quasi-perpendicular, isotropic, and quasi-parallel). The aspect angle is $\phi_B = 90^\circ$ in all images, i.e. the ambient magnetic field is perpendicular to the LoS; the angle $\phi_{\nabla B}$ is 0° for run Grad-BZ-g1 and 90° for Grad-BX-g1.

⁶ Note that $E_{f,\parallel}$ depends on the magnetic field strength at parallel shock which is different in the three configurations of unperturbed ISMF explored here due to the magnetic field gradient (in all the cases, we fix the magnetic field strength at the center of the SN explosion).

⁷ In the loss-limited model $E_{\max} \propto B^{-1/2}$, see Sect. 3.1.

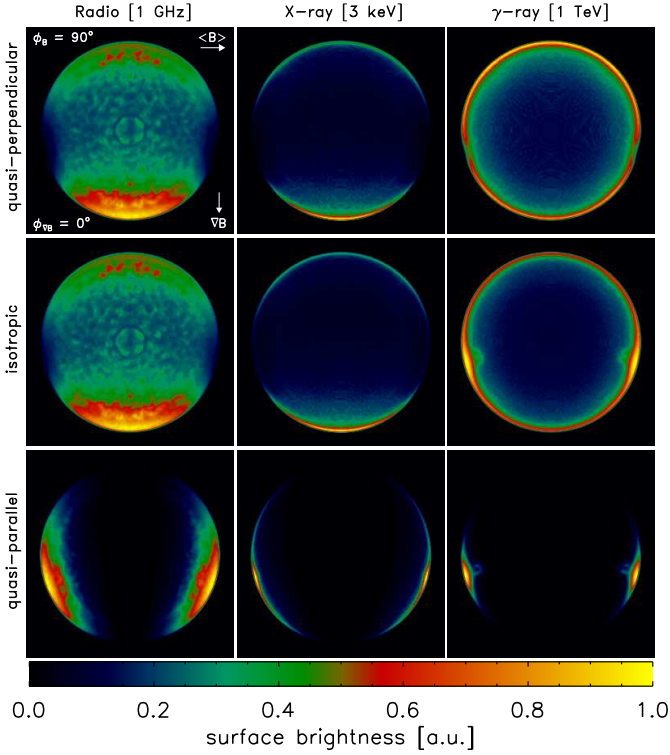


Figure 4. Maps of synchrotron radio (left), X-ray (center), and IC γ -ray (right) surface brightness (normalized to the maximum of each map) at $t = 1$ kyr synthesized from run Grad-BZ-g1, assuming randomized internal magnetic field. The relevant angles are $\phi_B = 90^\circ$ and $\phi_{\nabla B} = 0^\circ$. The figure shows the quasi-perpendicular (top), isotropic (middle), and quasi-parallel (bottom) particle injection models. The adiabatic index is $\gamma = 5/3$. The average ambient magnetic field is along the horizontal axis; the gradient of magnetic field strength is along the vertical axis.

The main factors affecting the azimuthal variations of surface brightness are the variations of: injection efficiency $f_\zeta(\Theta_o)$ and magnetic field $B_s(\Theta_o)$ in the radio band; $f_\zeta(\Theta_o)$, $B_s(\Theta_o)$ and maximum energy $E_{\max}(\Theta_o)$ in the X-ray band; $f_\zeta(\Theta_o)$ and $E_{\max}(\Theta_o)$ in the IC γ -ray band. Therefore, the morphology of the remnant in the three bands can differ considerably in appearance. In the radio and in the X-ray band, the remnant shows two lobes located at perpendicular shocks in the quasi-perpendicular and isotropic (i.e. where the magnetic field is larger), and at parallel shocks in the quasi-parallel model (i.e. where emitting electrons reside). The lobes are much thinner in X-rays than in radio because of the large radiative losses at the highest energies that make the X-ray emission dominated by radii closest to the shock. In the γ -ray band, the remnant morphology changes significantly in the three injection models: it is almost ring-like (with two faint minima at parallel shocks) when the injection is quasi-perpendicular; the morphology shows two lobes located at parallel shocks when the injection is isotropic, at variance with the lobes in radio and X-rays that are located at perpendicular-shocks (i.e. bright γ -ray lobes correspond to dark radio and X-ray areas); the morphology is characterized by two narrow bright lobes almost superimposed to those in radio and X-rays when the injection is quasi-parallel. A ring-like γ -ray morphology is compatible with those found by HESS in the SNRs RX J1713.7-3946 (Aharonian et al. 2006) and RX J0852.0-4622 (Vela Jr.; Aharonian et al. 2007b) where γ -rays are detected virtually throughout the whole remnant and the emis-

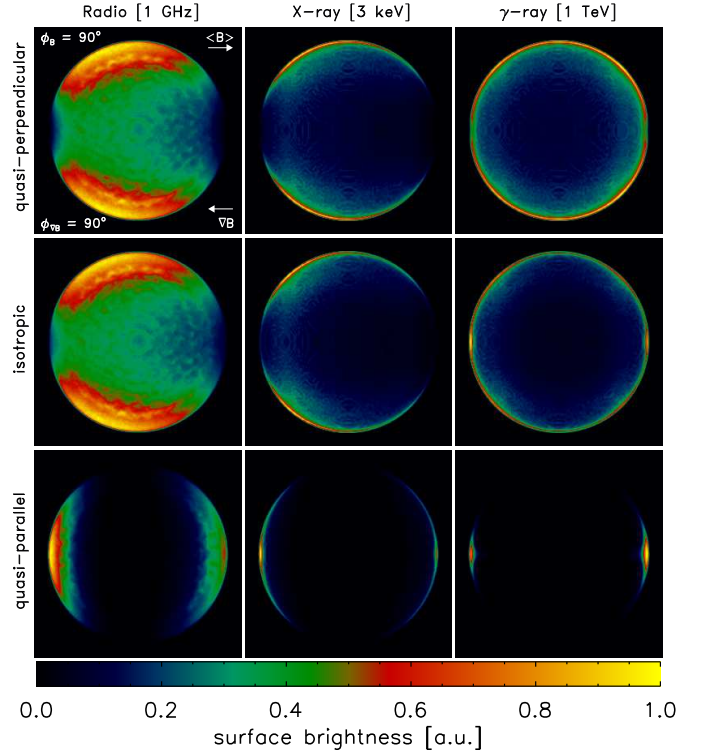


Figure 5. As in Fig. 4 for run Grad-BX-g1. Both the average ambient magnetic field and the gradient of magnetic field strength are along the horizontal axis. The relevant angles are $\phi_B = 90^\circ$ and $\phi_{\nabla B} = 90^\circ$.

sion is found to resemble a shell structure. On the other hand, the bipolar γ -ray morphology of SN 1006 revealed by HESS (Acero et al. 2010), with the bright lobes strongly correlated with non-thermal X-rays, may be easily reproduced in the polar-caps scenario (quasi-parallel injection).

The effects of the non-uniform ISMF on the remnant morphology in the X-ray band are similar to those discussed in Paper I for the radio band: remnants with two non-thermal X-ray lobes of different brightness (upper left panel in Fig. 4 and lower left panel in Fig. 5) are produced if a gradient of ambient magnetic field strength is perpendicular to the lobes; remnants with converging similar non-thermal X-ray lobes (lower left panel in Fig. 4 and upper right panel in Fig. 5) are produced if the gradient runs between the two lobes. Analogous asymmetries are found in the γ -ray morphology of the remnant although the degree of asymmetry is less evident. Note however that, in the case of isotropic injection, the γ -ray lobes are converging on one side when radio and X-ray lobes are characterized by different brightness (see Fig. 4). This is the consequence of the “limb-inverse” property in γ -rays⁸ (Petruck et al. 2009a). In general, this property is valid not only in the case of isotropic injection; this type of injection is just the more prominent case. In fact, the critical quantities determining the “limb-inverse” property are the contrasts between electron injection, ISMF, and model of E_{\max} . For

⁸ The “limb-inverse” property in γ -rays is determined for isotropic injection because the magnetic field affects the downstream distribution of IC γ -ray emitting electrons which is steeper where the magnetic field is stronger. The reader is referred to Petruck et al. (2009a) for more details.

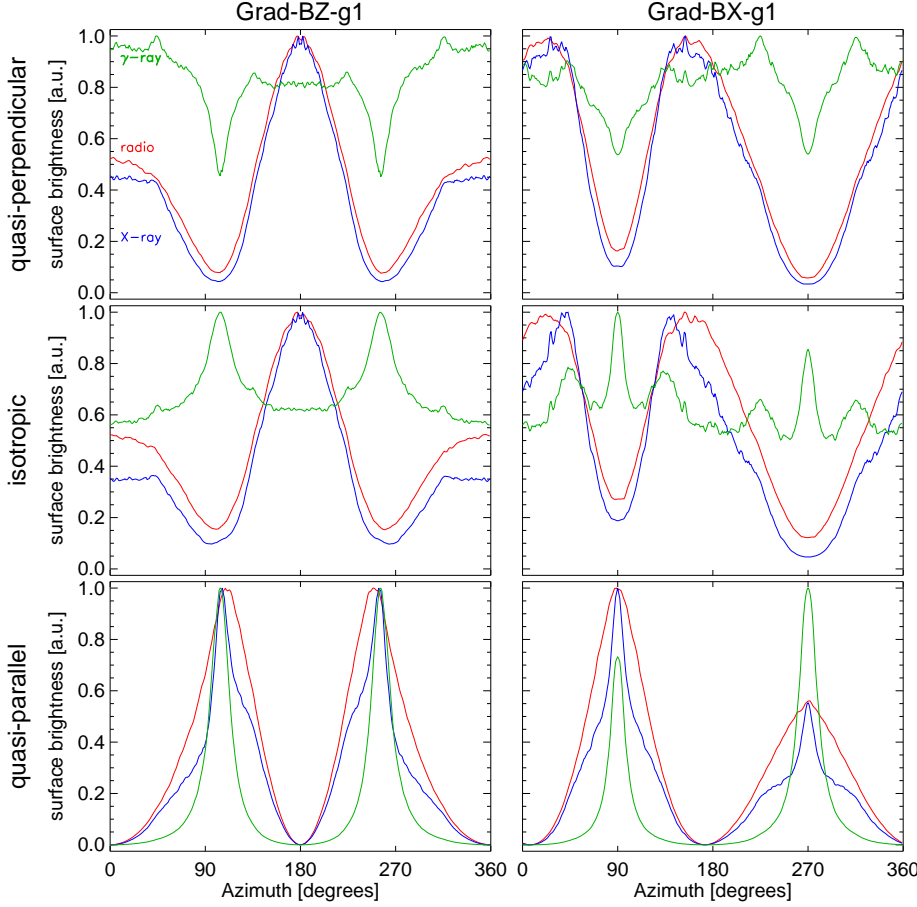


Figure 6. Azimuthal profiles of the synchrotron radio (red), X-ray (blue), and IC γ -ray (green) surface brightness synthesized from runs Grad-BZ-g1 (left; the relevant angles are $\phi_B = 90^\circ$ and $\phi_{VB} = 0^\circ$) and Grad-BX-g1 (right; $\phi_B = 90^\circ$ and $\phi_{VB} = 90^\circ$), assuming quasi-perpendicular (top), isotropic (middle), and quasi-parallel (bottom) injection models. The adiabatic index is $\gamma = 5/3$. The azimuth is measured counterclockwise from the north (see Figs. 4 and 5). The corresponding azimuthal profiles of E_{\max} used to derive the curves of this figure are in Fig. 3.

instance, in the case of uniform ISMF, the azimuthal contrast in IC γ -ray brightness is roughly

$$\begin{aligned} \frac{S_{\parallel}}{S_{\perp}} &\propto \frac{\text{injection}_{\parallel}}{\text{injection}_{\perp}} \exp \left[-E_m \left(\frac{1}{E_{\max,\parallel}} - \frac{1}{E_{\max,\perp}} \right) \right] = \\ &= \frac{\text{injection}_{\parallel}}{\text{injection}_{\perp}} \exp \left[-\frac{E_m}{E_{\max,\parallel}} \left(1 - \frac{E_{\max,\parallel}}{E_{\max,\perp}} \right) \right] \end{aligned} \quad (21)$$

where E_m is the electron energy which gives the maximum contribution to IC emission at a considered frequency and subscripts refer to positions along the limb where the ambient magnetic field is either parallel (\parallel) or perpendicular (\perp) to the shock normal. Even in the case of quasi-parallel injection ($\text{injection}_{\parallel}/\text{injection}_{\perp} > 1$), the contrast S_{\parallel}/S_{\perp} depends on the contrast of E_{\max} : the ratio $C_{\max} = E_{\max,\parallel}/E_{\max,\perp}$ may lead to an exponential term either > 1 or < 1 , leading to S_{\parallel}/S_{\perp} either > 1 or < 1 .

Another interesting feature characterizing the IC γ -ray morphology of the remnant is the inversion of the asymmetry when the two lobes have different brightness (i.e. a gradient of magnetic field strength is perpendicular to the lobes). This feature is evident in the upper panels of Fig. 4 and in the lower panels of Fig. 5: the brightest γ -ray lobe is located where both the radio and the X-ray lobes are fainter. As discussed in detail below in Sect. 4.4, this is due to the fact that, in the synthetic images presented in this section, E_{\max} depends inversely on the pre-shock ambient magnetic field strength (see Eq. 10 and Fig. 3) and its contrast is $C_{\max} > 1$.

Figure 6 shows the azimuthal profiles of the synchrotron radio, X-ray, and IC γ -ray surface brightness synthesized from

runs Grad-BZ-g1 and Grad-BX-g1 for the three injection models when the relevant angles are $\phi_B = 90^\circ$ and $\phi_{VB} = 0^\circ$ for run Grad-BZ-g1 and $\phi_{VB} = 90^\circ$ for Grad-BX-g1. In the quasi-parallel scenario, the non-thermal lobes are rather narrow azimuthally. Note the “limb-inverse” property in γ -rays for isotropic injection as discussed by Petruk et al. (2009a). Note also the “asymmetry-inverse” property in γ -rays when the two lobes have different brightness. In general we find that the degree of asymmetry (whatever the pattern of asymmetry – either different brightness or convergence of the lobes – is) induced by ∇B in the remnant morphology is different in the three bands (see Sect. 4.5 for a discussion on the degree of asymmetry of the remnant in the different bands); in particular, the IC γ -ray emission appears to be the less sensitive to the gradient.

Useful parameters to quantify the degree of asymmetry of the remnant are those defined in Paper I: the azimuthal intensity ratio $R_{\max} \geq 1$, i.e. the ratio of the maxima of intensity of the two lobes as derived from the azimuthal intensity profiles (a measure of different brightness of the lobes; $R_{\max} > 1$ in case of asymmetry), and the azimuthal distance θ_D , i.e. the distance in deg of the two maxima (a measure of the convergence of the lobes; $\theta_D < 180^\circ$ in case of asymmetry). For instance, in the case of quasi-parallel injection in Fig. 6 (lower panels), we find that the azimuthal distance θ_D ranges from 148° in γ -rays and X-rays to 134° in radio for run Grad-BZ-g1, and the azimuthal intensity ratio R_{\max} ranges from 1.4 in γ -rays to 1.8 in radio and X-rays for run Grad-BX-g1.

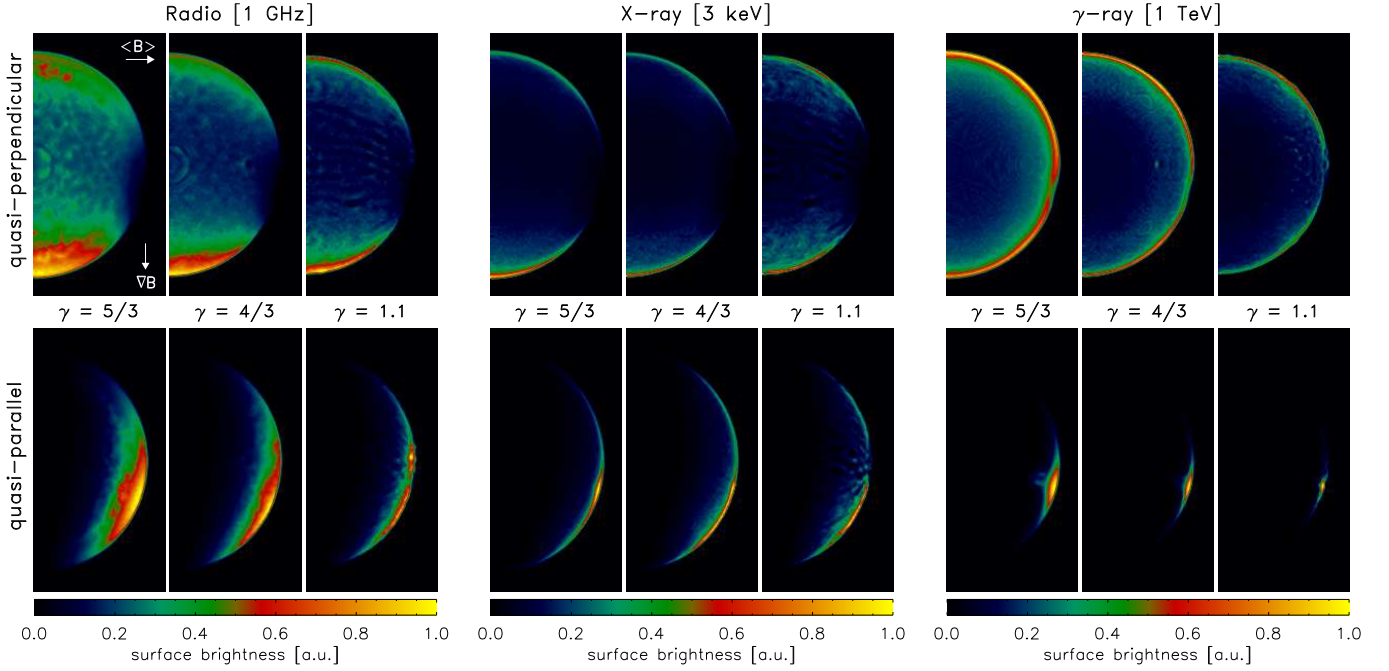


Figure 7. As in Fig. 4 for runs Grad-BZ-g1 ($\gamma = 5/3$), Grad-BZ-g2 ($\gamma = 4/3$), and Grad-BZ-g3 ($\gamma = 1.1$). The figure shows synchrotron radio (left panels), X-ray (center panels) and IC γ -ray (right panels), assuming either quasi-perpendicular (upper panels) or quasi-parallel (lower panels) injection models. Each panel shows only one half of the remnant which is symmetric with respect to the vertical axis.

4.3. Dependence on the adiabatic index

Petruk et al. (2010b, submitted to MNRAS) analyzed the effect of γ on non-thermal images of SNR expanding through homogeneous ISM and uniform ISMF. They showed that reducing the value of γ , the synchrotron brightness of the remnant is modified by increased radiative losses of emitting electrons, due to increased compression of \mathbf{B} , which results in thinner radial profiles of brightness. Figure 7 shows maps of synchrotron radio, X-ray, and IC γ -ray emission for the case of ISMF characterized by a gradient of field strength perpendicular to the average magnetic field and different values of the adiabatic index γ (runs Grad-BZ-g1, Grad-BZ-g2, and Grad-BZ-g3). As expected, the index γ determines both the shock compression ratio σ and the distance of the contact discontinuity from the blast wave position D_{cd} (see also Fig 1 in the case of uniform ISMF): the smaller γ , the larger σ (and the larger the radiative losses of emitting electrons) and the shorter D_{cd} . As shown in the figure, the main effect of smaller γ is to make thinner the lobes emitting synchrotron emission in the three bands. In particular, in the extreme case of $\gamma = 1.1$, the lobes are so thin that they are largely perturbed by the hydrodynamic instabilities forming at the contact discontinuity, the typical size of the instabilities being comparable with D_{cd} . The adiabatic index slightly influences also the azimuthal thickness of the lobes, especially in the quasi-parallel case: the smaller γ , the narrower this thickness. Nevertheless, the adiabatic index does not change significantly neither the degree nor the pattern of asymmetry of the remnant morphology caused by the gradient of magnetic field strength.

4.4. Dependence on the maximum energy

In Sect. 4.2, we have presented the remnant morphology for a reference case for which E_{\max} calculated at each point as

$E_{\max} = \min[E_{\max,1}, E_{\max,2}, E_{\max,3}]$ (where indexes 1, 2, 3 correspond respectively to loss-limited, time-limited and escape-limited models; see discussion in Sect. 3.1) is characterized by a contrast $C_{\max} > 1$. Here we generalize our study by considering some arbitrary smooth variations of E_{\max} versus obliquity with the goal to see how different trends in the obliquity dependence of E_{\max} influence the visible morphology of SNRs. That is, we do not use here the particular prescriptions for E_{\max} from Sect. 3.1, but simply assume that the acceleration physics is able to operate to produce an E_{\max} of prescribed properties. In fact, the model of E_{\max} and its obliquity dependence may affect both the degree and the pattern of asymmetry of the remnant morphology. A critical point is if E_{\max} depends directly or inversely on the magnetic field strength. Moreover the remnant morphology in the various bands can be characterized by different features if the contrast C_{\max} is either > 1 or < 1 . As an example, Fig. 8 shows the azimuthal profiles of three arbitrary models of E_{\max} characterized by different dependencies on the obliquity angle: in model A, $E_{\max} \propto B$ and its contrast is $C_{\max} < 1$ (solid line); in model B, $E_{\max} \propto B^{-1/2}$ and $C_{\max} > 1$ (dotted); in model C, $E_{\max} \propto B$ and C_{\max} is < 1 on the side of the remnant with the strongest ISMF strength and > 1 on the side with the lowest field strength. Note that model B coincides with the model of E_{\max} computed for the reference case in Sect. 4.2 (see Fig. 3). The asymmetries on the profiles of E_{\max} are introduced by the gradient of ambient magnetic field, as discussed in Sect. 4.2.

From the models of E_{\max} shown in Fig. 8, we synthesized maps of synchrotron radio, X-ray, and IC γ -ray emission. We find that the remnant morphology is very sensitive to the model of E_{\max} when a gradient of magnetic field strength is perpendicular to the lobes and the latter are characterized by different brightness. The asymmetry between the two lobes can be reduced in the X-ray band or even inverted in the IC γ -ray band when E_{\max} depends inversely on the pre-shock ambient mag-

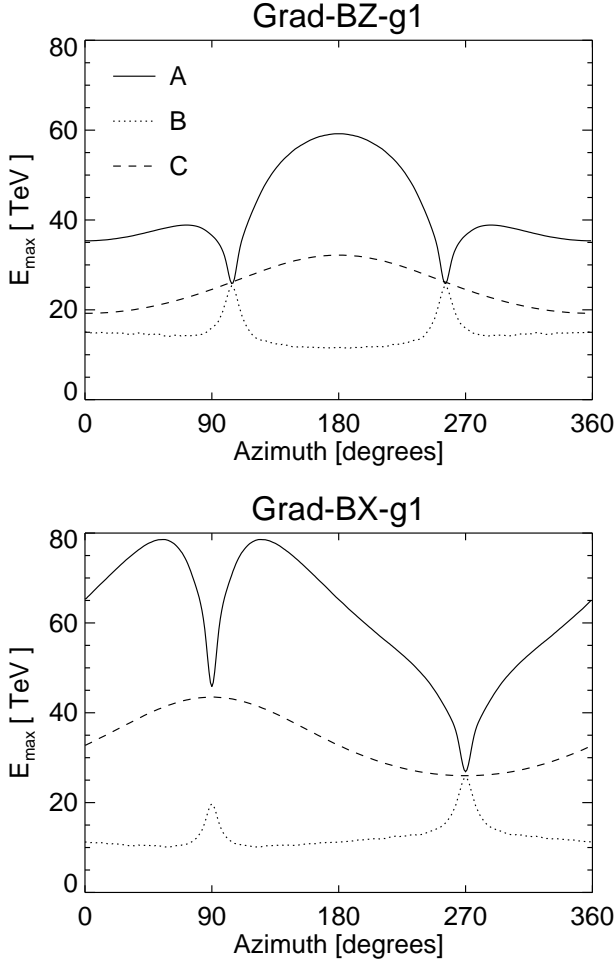


Figure 8. As in Fig. 3 but for three different models of E_{\max} characterized by smooth variations with the obliquity angle. The azimuth is measured counterclockwise from the “north” of the remnant. The shock is parallel around 90° and 270° and perpendicular around 180° and 360° . In model A (solid line), the contrast of E_{\max} is $C_{\max} < 1$, in model B (dotted) $C_{\max} > 1$, and in model C (dashed) C_{\max} is < 1 on the side of the remnant with the strongest ISMF strength and > 1 on the side with the lowest field strength (see text).

netic field strength, namely in the case of model B in Fig. 8. In particular, this model of E_{\max} leads to the “asymmetry-inverse” property in γ -rays already discussed in Sect. 4.2 for our reference case (see upper right panel in Fig. 4 and lower right panel in 5).

Fig. 9 shows the azimuthal profiles of the IC γ -ray surface brightness synthesized from runs Grad-BZ-g1 and Grad-BX-g1 when the lobes have different brightness, for the three models of E_{\max} reported in Fig. 8. In the case of the IC surface brightness, the asymmetry-inverse property is evident when E_{\max} is the largest where the magnetic field strength is the lowest (see model B in Fig. 8). This is due to the fact that the IC emissivity $i(\epsilon)$ weakly depends on B . In the case of the non-thermal X-ray surface brightness, the inverse dependence of E_{\max} on B partially contrast the dependence of the non-thermal X-ray $i(\epsilon)$ on B , reducing the degree of asymmetry between the lobes. It is worth to note that, if E_{\max} is high enough in regions with weak magnetic field, than the inversion of asymmetry may be present even in the X-ray band.

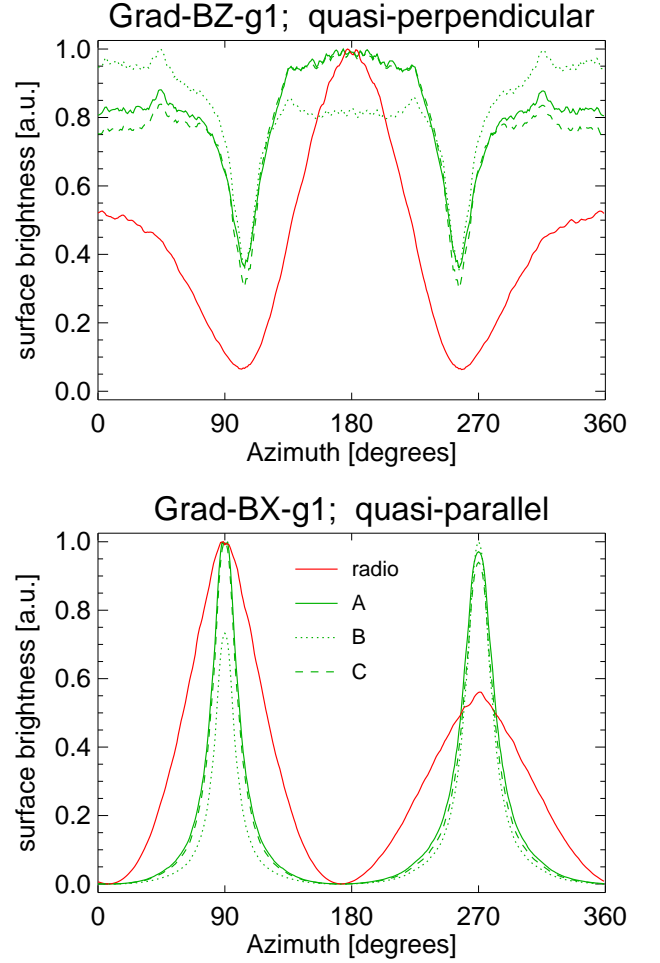


Figure 9. Azimuthal profiles of the synchrotron radio (red lines) and IC γ -ray surface brightness (green) synthesized from run Grad-BZ-g1 with quasi-perpendicular injection (top), and from run Grad-BX-g1 with quasi-parallel injection (bottom); the aspect angle is $\phi_B = 90^\circ$. The models of E_{\max} adopted to synthesize the non-thermal emission are those shown in Fig. 8: model A (solid line), B (dotted) and C (dashed).

On the other hand, we also found that when the non-uniform ISMF leads to non-thermal lobes converging on one side (i.e. when a gradient of ISMF is running between the lobes) the model of E_{\max} does not affect significantly the degree and the pattern of asymmetry of the remnant morphology (see lower panel in Fig. 9).

4.5. Dependence on the orientation of ISMF gradient

As expected, the degree of asymmetry of the remnant morphology depends on the orientation of ∇B with respect to the plane of the sky. In the case of run Grad-BZ-g1, Fig. 10 shows the azimuthal intensity ratio R_{\max} and the azimuthal distance θ_D vs. the angle $\phi_{\nabla B}$, for an aspect angle $\phi_B = 90^\circ$ and for different trends in the obliquity dependence of E_{\max} (exploring the contrasts C_{\max} either > 1 or < 1 ; see Fig. 8). The asymmetries are the largest when ∇B lies in the plane of the sky (i.e. $\phi_{\nabla B} = 0^\circ$), whereas no asymmetries are present when ∇B is along the LoS (i.e. $\phi_{\nabla B} = 90^\circ$). In all the intermediate cases, the degree of asymmetry is determined by the component of ∇B lying in the plane of the sky. Note that the remnant morphology

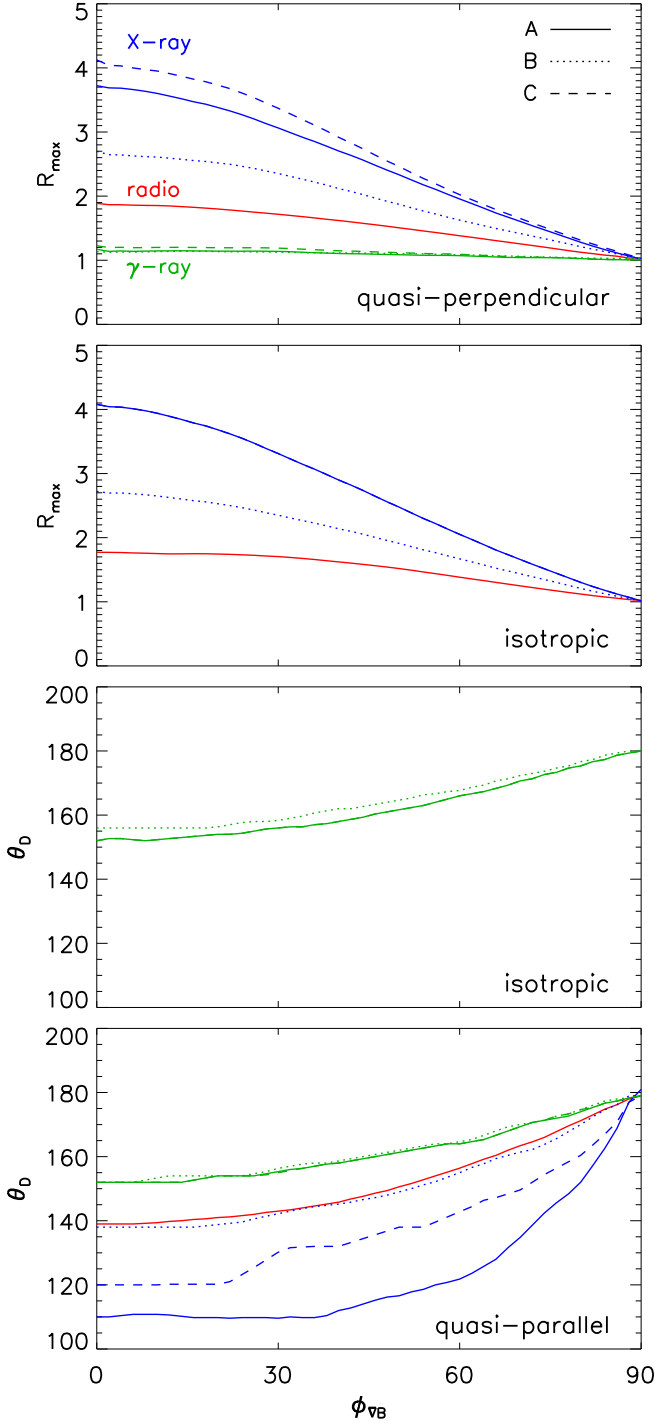


Figure 10. Azimuthal intensity ratio R_{\max} (i.e. the ratio of the maxima of intensity of the two lobes around the shell) and azimuthal distance θ_D (i.e. the distance in deg of the two maxima of intensity around the shell) vs. the angle between ∇B and the vertical line passing through the remnant center $\phi_{\nabla B}$, for an aspect angle $\phi_B = 90^\circ$, and for the three models of E_{\max} shown in Fig. 8. The run is Grad-BZ-g1. For isotropic injection, curves for R_{\max} in γ -rays and curves for θ_D in radio and X-rays are not shown, the values being $R_{\max} = 1$ and $\theta_D = 180^\circ$, respectively at all $\phi_{\nabla B}$.

shows only one kind of asymmetry when the injection is quasi-perpendicular or quasi-parallel and the aspect angle is $\phi_B = 90^\circ$.

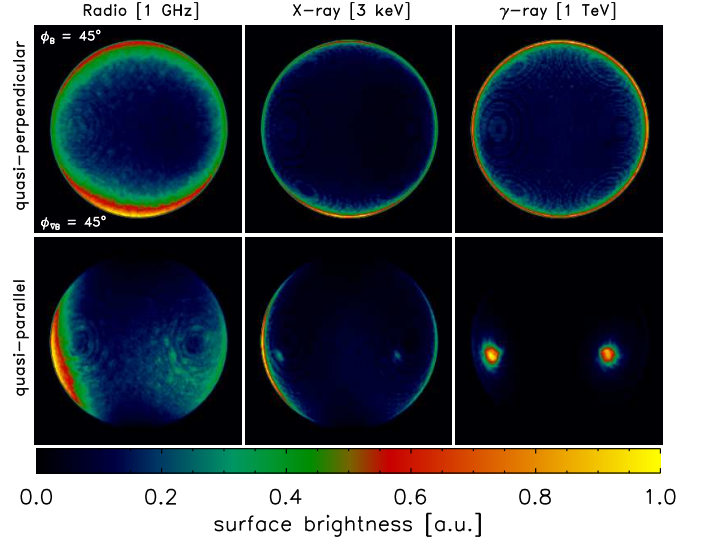


Figure 11. Maps of synchrotron radio (left), X-ray (center), and IC γ -ray (right) surface brightness synthesized from run Grad-BZ-g1, assuming quasi-perpendicular (top), and quasi-parallel (bottom) injection models. The adiabatic index is $\gamma = 4/3$. The maps have been synthesized adopting model B of E_{\max} shown in Fig. 8. The relevant angles are $\phi_B = 45^\circ$ and $\phi_{\nabla B} = 45^\circ$. The angle between $\langle \mathbf{B} \rangle$ and $\nabla|\langle \mathbf{B} \rangle|$ is 90° .

On the other hand, the lobes have different brightness in radio and non-thermal X-rays and are converging in IC γ -rays when the injection is isotropic due to the "limb-inverse" property.

In general we find that the degree of asymmetry (whatever the pattern of asymmetry – either different brightness or convergence of the lobes – is) induced by ∇B in the remnant morphology is different in the three bands: the non-thermal X-ray (IC γ -ray) emission appears to be the most (less) sensitive to the gradient. This happens because the emissivity $i(\epsilon)$ depends directly on the magnetic field strength (see Eq. 3) only in the synchrotron emission process (no in the IC process). Consequently, the IC γ -ray emission shows a weaker dependence on the ∇B . In fact, IC brightness depends on \mathbf{B} indirectly, through radiative losses of electrons: larger \mathbf{B} induces decrease of the number of electrons emitting IC γ -rays. Note that the sensitivity on ∇B depends also on the energy of photons, and on the reduced fiducial energy E_f , which is the measure of efficiency of the role of radiative losses in modification of downstream evolution of emitting electrons. Note also that the degree of asymmetry of the remnant morphology can be significantly reduced when the E_{\max} contrast is $C_{\max} > 1$ (e.g. model B in Fig. 8). In fact, the asymmetry reduction is mainly due to the dependency of E_{\max} on the magnetic field strength. As discussed in Sect. 4.4, the asymmetries in the remnant morphology can be reduced or even inverted when E_{\max} depends inversely on the pre-shock ambient magnetic field strength (which is the case in model B).

When the ∇B is not aligned with the average ambient magnetic field (for instance in the case of run Grad-BZ-g1), the projection of the ∇B in the plane of the sky has (for generic values of ϕ_B and $\phi_{\nabla B}$) a component perpendicular to the projected lobes and one running between them. In this case both kind of asymmetries (lobes converging on one side and with different brightness) are expected in the remnant morphology. As an example, Fig. 11 shows the synchrotron radio, X-ray, and IC γ -ray images synthesized from run Grad-BZ-g1, for different injection models. The relevant angles are $\phi_B = 45^\circ$ and $\phi_{\nabla B} = 45^\circ$.

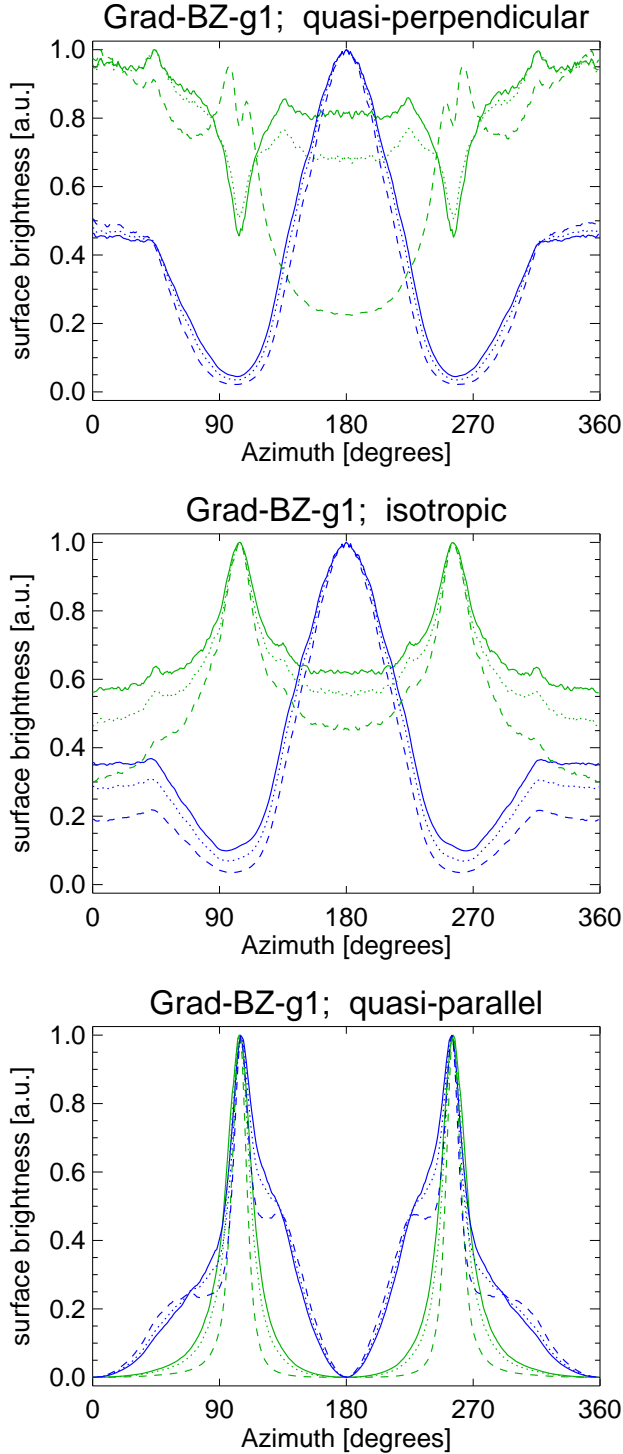


Figure 12. As in Fig. 6, for the azimuthal profiles of the synchrotron X-ray (blue) and IC γ -ray (green) surface brightness synthesized from run Grad-BZ-g1, for $E_{\max,||} = 26$ TeV (solid), 5 TeV (dotted), and 1 TeV (dashed). The surface brightness is synthesized deriving the maximum energy E_{\max} as described in Sect. 4.2; its azimuthal profile is similar to that shown in Fig. 3 (solid line) but with a different value of $E_{\max,||}$. In all the cases, $E_{f,||} = 12$ TeV.

4.6. Dependence on the value of $E_{\max,||}$

The calculations presented above assume the free parameter $E_{\max,||} = 26$ TeV larger than the fiducial energy $E_{f,||} = 12$ TeV,

implying that the electron energy losses are mainly due to radiative losses (see Sect. 3.2). The variation of the energy distribution $N(E, a, t)$ of electrons in Eq. 19 is influenced by radiative losses of electrons that are, therefore, important for the surface brightness distribution of the remnant in X-rays and γ -rays. Consequently, the choice of $E_{\max,||}$ may influence both the degree and the pattern of asymmetry of the remnant morphology. In particular, in cases with $E_{\max} < E_f$, we expect that $\mathcal{E}_{\text{rad}} \rightarrow 1$ (i.e. electrons have negligible radiative losses) and the electron energy losses are mainly due to adiabatic expansion. This issue is investigated by considering the reference case discussed in Sect. 4.2 and two additional cases for which $E_{\max} < E_f$, namely $E_{\max,||} = 5$ TeV and $E_{\max,||} = 1$ TeV. Figure 12 shows the azimuthal profiles of the synchrotron X-ray and IC γ -ray surface brightness synthesized from run Grad-BZ-g1, for these values of $E_{\max,||}$ together with the case with $E_{\max,||} = 26$ TeV (the reference case). Note that E_{\max} is calculated at each point of the domain as described in Sect. 4.2 but for different values of $E_{\max,||}$ (see also Sect. 3.1). The figure shows that for decreasing values of $E_{\max,||}$, the contrast of emission increases, the effect being the largest for IC γ -ray emission than for synchrotron X-rays. Nevertheless, the degree and the pattern of asymmetry of the remnant morphology induced by the gradient of ISMF are only slightly influenced by the value of $E_{\max,||}$.

5. Summary and conclusions

We developed a numerical code (REMLIGHT) to synthesize the synchrotron radio, X-ray, and IC γ -ray emission from MHD simulations, in the general case of a remnant expanding through a non-uniform ISM and/or a non-uniform ISMF. As a first application of REMLIGHT, we coupled the synthesis code to the MHD model discussed in Paper I (extended to include an approximate treatment of upstream magnetic field amplification and the effect of shock modification due to back reaction of accelerated CRs) and investigated the effects of a non-uniform ISMF on the remnant morphology in the X-ray and γ -ray bands. Our findings lead to several conclusions:

- A gradient of ISMF strength induces asymmetries in both the X-ray and γ -ray morphology of the remnant if the gradient has a component perpendicular to the LoS. In general, the asymmetries are analogous to those found in Paper I in the radio band, independently from the models of electron injection and of maximum energy of electrons accelerated by the shock. In the γ -ray band, the asymmetry in the remnant morphology is inverted with respect to those in the radio and X-ray bands if the model of E_{\max} depends inversely on the pre-shock magnetic field strength and its contrast is $C_{\max} > 1$ (e.g. model B in Fig. 8): the brightest γ -ray lobe is located where both the radio and the X-ray lobes are the faintest.
- The non-thermal lobes are characterized by different brightness when a gradient of ISMF strength is perpendicular to the lobes; they are converging on one side when a gradient of ISMF is running between them. In the general case of a gradient with components parallel and perpendicular to the lobes, both kinds of asymmetry may characterize the remnant morphology.
- The non-thermal X-ray emission is confined in very thin limbs because of the large radiative losses at high energy and, in general, is the most sensitive to non-uniform ISMF. In fact the remnant morphology in this band shows the highest degree of asymmetry among the images synthesized in the three bands of interest (i.e. radio, X-ray, and γ -ray), ex-

cept when E_{\max} depends inversely on the pre-shock magnetic field strength. In the latter case, the asymmetries in the X-ray band can be significantly reduced.

- The IC γ -ray emission is weakly sensitive to the non-uniform ISMF, the degree of asymmetry being the lowest in the three bands considered. The remnant morphology is almost ring-like for quasi-perpendicular injection, shows the “limb-inverse” property discussed by Petruk et al. (2009a) for isotropic injection (i.e. bright γ -ray lobes correspond to dark radio and X-ray areas), and is bilateral for quasi-parallel injection. The “limb-inverse” property implies, for instance, that γ -ray lobes are symmetric and converging on one side when radio and X-ray lobes have different brightness (see Fig. 4). In case E_{\max} depends inversely on the pre-shock ambient magnetic field strength, the asymmetries in the IC γ -ray morphology can be inverted; for instance, brightest γ -ray lobes can be located where both radio and X-ray lobes are fainter. Note that the γ -ray morphology of the SNRs RX J1713.7-3946 (Aharonian et al. 2006) and RX J0852.0-4622 (Aharonian et al. 2007b) could be reproduced in the equatorial-belt scenario (the injection is either quasi-perpendicular or isotropic), whereas the morphology of SN 1006 (Acero et al. 2010) is compatible with that predicted in the polar-caps scenario (quasi-parallel injection).

Note that, although the MHD model presented here does not include self-consistently shock modification and magnetic field amplification, we adopted an approximate treatment of both processes. Magnetic field amplification could result from streaming instability excited by the accelerated particles upstream of the shock or, alternatively, the magnetic fields could be amplified in a purely hydrodynamic way in the downstream plasma (Giacalone & Jokipii 2007). In both cases, the shock is expected to be modified due to the dynamical reaction of the amplified magnetic field (see, for instance, Ferrand et al. 2010 for an hydrodynamic model including back-reaction of accelerated CRs). In this paper, we approach the effect of shock modification by considering different values of the adiabatic index γ (namely, 5/3, 4/3, 1.1) and the effect of upstream magnetic field amplification by considering the ambient magnetic field strength enhanced by $\times 10$ in the neighborhoods of the remnant (the unperturbed field strength commonly expected is a few μG). The main effect of γ is to change the compression ratio of the shock and the distance of the contact discontinuity from the blast wave position. In the simplest case considered here, namely the modification on γ and the upstream magnetic field amplification are both isotropic with no dependence on the obliquity angle, we found that the modified γ and the amplified field influence mainly the absolute values of non-thermal emission but not the large scale morphology of the remnant and the pattern of asymmetries induced by a non-uniform ISMF. The results presented here, therefore, are only valid in this case. Conversely, we expect a significant effect of the modified γ as well as of the amplified field on the remnant morphology if the shock modification and/or upstream magnetic field amplification depend on the obliquity. This issue deserves further investigation in future studies.

It is worth to emphasize that the calculations provided in this paper (and implemented in the *REMLIGHT* code) to synthesize the non-thermal emission from MHD simulations consider a generic adiabatic index γ . The *REMLIGHT* code therefore can be easily coupled with a model including the back-reaction of accelerated CRs and synthesize the non-thermal emission consistently if the

value of an “effective” γ is provided in each point of the spatial domain⁹.

Note also that the MHD model adopted here follows the evolution of the remnant during the adiabatic phase and, therefore, its applicability is limited to this evolutionary stage. In the radiative phase, the high degree of compression suggested by radiative shocks leads to increase in the synchrotron emission brightness due to compression of ambient magnetic field and electrons. Since our model neglects the radiative cooling of the shocked gas, it is limited to compression ratios derived from γ and, therefore, it is not able to simulate this mechanism of limb brightening. Nevertheless, the model is appropriate to describe young SNRs that are those from which non-thermal emission is commonly detected.

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⁹ See, for instance, Ferrand et al. (2010) for an hydrodynamic model calculating the “effective” γ in each point of the spatial domain (see also Ellison et al. 2004)

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Appendix A: Calculation of integral in Eq. 14

The function $\mathcal{I}(a, t)$ in Eq. 14 is expressed as (Reynolds 1998)

$$\mathcal{I}(a, t) = \sigma_B^2 \int_{t_i}^t \frac{B_{\text{eff}}(a, t')^2}{B_{\text{eff},s}(t')^2} \left(\frac{\rho(a, t')}{\rho(a, t)} \right)^{1/3} \frac{dt'}{t'}, \quad (\text{A.1})$$

where $B_{\text{eff}}^2 = B^2 + B_{\text{CMB}}^2$ is the “effective” magnetic field introduced to account for the energy losses of electrons due to IC scatterings on the photons of CMB.

The integral (A.1) is rather CPU consuming because it requires to know, with high enough time resolution, the history of each parcel of gas inside the SNR since its shocking time. To reduce the computational cost, we calculate it approximately, changing integration on dt' to $dR' = V_{\text{sh}}(t')dt'$, where R and V_{sh} are the shock position and velocity, respectively, and using some MHD properties of the fluid.

We calculate $\mathcal{I}(a, t)$ using an analytic description of mass density and magnetic field evolution inside the SNR which expands through a non-uniform ISM and/or ISMF. The continuity equation $\rho_o(a)a^2 da = \rho(a)r^2 dr$ results in

$$\rho(a, t) = \rho_o(a) \left(\frac{a}{r(a, t)} \right)^2 r_a(a, t)^{-1} \quad (\text{A.2})$$

where $r_a(a, t)$ is the derivative of $r(a, t)$ with respect to a ; the density term in Eq. A.1 is

$$\frac{\rho(a, t')}{\rho(a, t)} = \frac{r(a, t)^2}{r(a, t')^2} \frac{r_a(a, t)}{r_a(a, t')} \quad (\text{A.3})$$

The magnetic field in Eq. A.1 can be expressed as $B(a, t)^2 = B_{\parallel}(a, t)^2 + B_{\perp}(a, t)^2$, where B_{\parallel} and B_{\perp} are the components of magnetic field parallel and perpendicular to the shock normal, respectively. These two components follow the magnetic flux conservation $B_{\parallel} d\sigma_S = \text{const}$, where $d\sigma_S$ is a surface element, and the flux-frozen condition $B_{\perp}(r) r dr = \text{const}$:

$$B_{\parallel}(a, t) = B_{\parallel,o}(a) \frac{a^2}{r(a, t)^2}, \quad (\text{A.4})$$

$$B_{\perp}(a, t) = B_{\perp,o}(a) \frac{r(a, t)}{a} \frac{\rho(a, t)}{\rho_o(a)} = B_{\perp,o}(a) \frac{a}{r(a, t) r_a(a, t)}. \quad (\text{A.5})$$

Thus, the magnetic field and the mass density in Eq. A.1 can be expressed through the relation $r(a, t)$ between Eulerian and Lagrangian coordinates of a parcel of gas and its derivative, $r_a(a, t)$. Considering that $r(a, t)$ and $r_a(a, t)$ can be expressed in terms of the dynamical characteristics of the shock (i.e. as $r(a, R)$ and $r_a(a, R)$), the integral A.1 may be calculated as follows:

$$\mathcal{I}(a, t) = \frac{\sigma_B^2}{t} \int_{R_1}^R \frac{B(a, R')^2}{B_s(R')^2} \left(\frac{r(a, R')^2 r_a(a, R)}{r(a, R')^2 r_a(a, R')} \right)^{1/3} \frac{dR'}{V_{\text{sh}}(R')}. \quad (\text{A.6})$$

Now, the relation $r(a, R)$ is approximated¹⁰, using the method described by Hnatyk & Petruk (1999):

$$\frac{r(a, R)}{R} = \left(\frac{a}{R} \right)^{\psi} (1 + a_1 v + a_2 v^2 + a_3 v^3 + a_4 v^4) \quad (\text{A.7})$$

where $v = (R - a)/R$ and $\psi = (\gamma - 1)/\gamma$. The parameters a_1 , a_2 , a_3 , and a_4 are expressed as:

$$a_1 = -r_{a,s} + \psi, \quad (\text{A.8})$$

$$a_2 = \frac{1}{2} (R r_{aa,s} - 2\psi r_{a,s} + \psi(\psi + 1)), \quad (\text{A.9})$$

$$a_3 = \frac{1}{6} (-R^2 r_{aaa,s} + 3\psi R r_{aa,s} - 3\psi(\psi + 1) r_{a,s} + \psi(\psi + 1)(\psi + 2)), \quad (\text{A.10})$$

$$a_4 = C - (1 + a_1 + a_2 + a_3), \quad (\text{A.11})$$

where C reflects the variation of $r(a)$ around the center of the SNR. We adopt $C = C_A$ where C_A is given by the self-similar Sedov solution for a spherical shock (for details see Sect. 4.3 and Appendix in Petruk 2000 and references therein):

$$C_A = \left[\frac{\gamma}{\gamma + 1} \bar{P}(0)^{-1/\gamma} \right]^{1/3}, \quad (\text{A.12})$$

$\bar{P}(0)$ is the plasma pressure at the center of the remnant divided by its post-shock value

$$\bar{P}(0) = \left(\frac{1}{2} \right)^{6/5} \left(\frac{\gamma + 1}{\gamma} \right)^{6/5 - \gamma/(2-\gamma)} \left(\frac{(2\gamma + 1)(\gamma + 1)}{\gamma(7 - \gamma)} \right)^{(-2 + 5/(2-\gamma))\zeta}, \quad (\text{A.13})$$

$$\zeta = \frac{\gamma + 1}{3(\gamma - 1) + 2} - \frac{2}{5} + \frac{\gamma - 1}{2\gamma + 1}. \quad (\text{A.14})$$

Thus, we derive $C_A(\gamma = 5/3) = 1.083$, $C_A(\gamma = 4/3) = 1.055$ and $C_A(\gamma = 1.1) = 1.021$. The expressions for the derivatives $r_{a,s}$, $r_{aa,s}$, $r_{aaa,s}$ in Eqs. A.8-A.10 as functions of R , \dot{R} , \ddot{R} and $R^{(3)}$ are given in Appendix A2 of Hnatyk & Petruk (1999).

Finally, we calculate $V_{\text{sh}}(R)$ in Eq. A.1 as well as \dot{R} and $R^{(3)}$, using the Hnatyk (1987) approximate analytical formula for the strong shock in a non-uniform medium (see also Sect. 2.1 in Hnatyk & Petruk 1999).

The integral $\mathcal{I}(a, t)$ can be calculated rather simply in the case of a SNR expanding through uniform ISM and ISMF. We therefore test our calculation of $\mathcal{I}(a, t)$ by comparing the approximate values derived from Eq. A.6 with the exact ones derived from the Sedov solution in the case of $\gamma = 5/3$. Figure A.1 compares the exact and approximate values of $\mathcal{I}(a, t)$ in the limits of parallel and perpendicular shocks. Note that the approximate values are very accurate at radii close to the shock front where most of the non-thermal emission originates.

¹⁰ The approximation (A.7) is developed to give exact values of derivatives up to the third order at the shock and to the first order at the center.

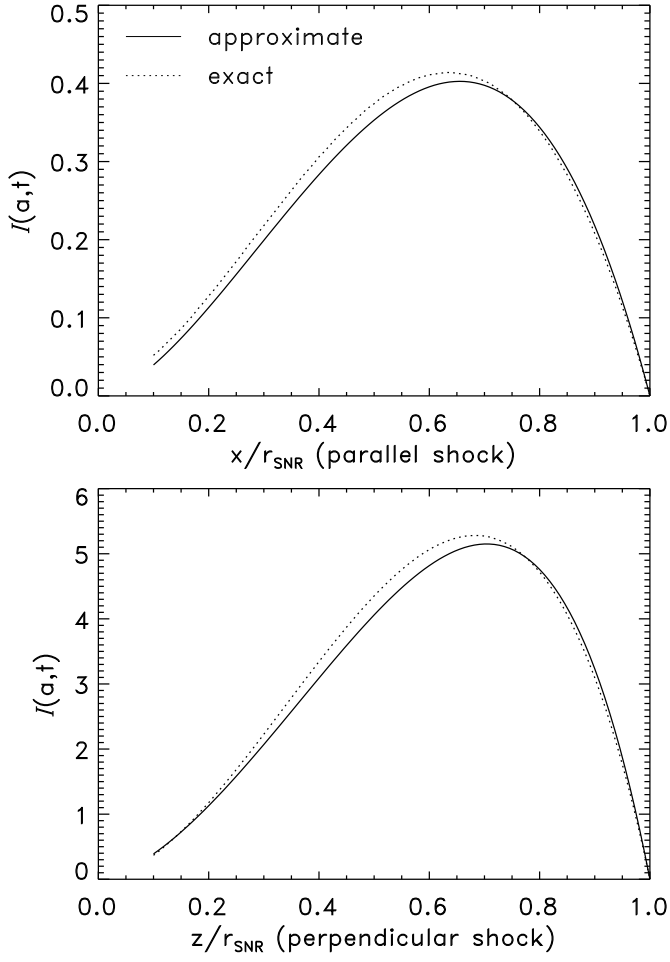


Figure A.1. Self-similar approximate and exact radial profiles of the integral $I(a)$ when the ambient magnetic field is either parallel (upper panel) or perpendicular (lower panel) to the shock normal, and $\gamma = 5/3$.